PERI AIGYPTON HAI
MATHAEMATIKAI PROTON
TECHNAI SYNESTAESAN
Aristotle, metaphysics, book 1, chapter 1

CORRECT METHOD OF RECKONING,
FOR GRASPING THE MEANING OF
THINGS AND KNOWING EVERYTHING THAT IS,
OBSCURITIES ... AND ALL SECRETS

Ahmes, opening lines of the
Rhind Mathematical Papyrus
(translated by Gay Robins & Charles Shute)

Rhind Mathematical Papyrus

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Inhaltsverzeichnis

How long are the diagonals of a square?

By drawing a grid and measuring the diagonals of various squares one may find the following numbers:

```
side  2     diagonal  a little less than  3
side  3     diagonal  a little more than  4
side  5     diagonal               about  7
side  5     diagonal  slightly more than  7
side  7     diagonal  slightly less than 10
side 12     diagonal        practically  17
side 12     diagonal        practically  17
side 17     diagonal        practically  24
side 29     diagonal        practically  41   and so on
```

These numbers generate a simple pattern (add a pair of numbers and you obtain the number below, double the first number of a line and you obtain the last number)

```
1       1       2
  2       3       4
    5       7      10
      12      17      24
        29      41      58
          70      99     140
            169     239     338
              408     577     816
                985    1393    ....
                  ....    ....    ....
```

If a square measures 70 by 70 royal cubits, the diagonals measure practically 99 royal cubits, and if a square measures 99 by 99 royal cubits, the diagonals measure practically 140 royal cubits. - How long is the diagonal of a square if the side measures 118 royal cubits or 826 palms? You may proceed as follows:

```
side  118  =  99  +  12  +  7
      140  +  17  + 10  =  167  diagonal
```

If the side of a square measures 118 royal cubits, the diagonal measures about 167 royal cubits (mistake 6.43 centimeters).

```
side  826  =  577  +  169  +  70  +  7  +  3
       816  +  239  +  99  + 10  +  4  =  1168
```

If the side of a square measures 118 royal cubits = 826 palms, the diagonal measures about 1168 palms or 166 cubits 6 palms (mistake 1 centimeter).

Divide 1393 by 985 and you obtain 1;24,51,10,3,2... in the sexagesimal number system of Babylon. Leave out the small numbers 3,2... and keep the value 1;24,51,10. This excellent value for the square root of 2 is found on the Babylonian clay tablet YBC 7289 dating from around 1650 BC.

Cube and equilateral triangle

Let me draw up an analogous number pattern:

```
 1        1        3
   2        4        6
   1        2        3
     3        5        9
       8       14       24
       4        7       12
         11       19       33
           30       52       90
           15       26       45
             41       71      123
               112      194      336
                56       97      168
                   153      265      459    (Archimedes)
                    418      724     1254
                    209      362      627
                      571      989     1713
                        1560     2702     ....
                         780     1351     ....     (Archimedes)
```

If a cube measures 41 by 41 by 41 fingers, the diagonals of the faces measure about 58 fingers (first number pattern) while the cubic diagonals measure about 71 fingers.

If the side of an equilateral triangle measures 194 fingers, its height measures practically 168 fingers or 24 palms or 6 royal cubits; the radius of the inscribed circle measures practically 56 fingers or 2 royal cubits, and the radius of the circumscribed circle 112 fingers or 4 royal cubits.

The side of an equilateral hexagon measures 10 royal cubits or 70 palms or 280 fingers. Calculate the diameter of the inscribed circle. Proceed as follows:

```
 side  280  =  194 + 56 + 19 + 15
       336 + 90 + 33 + 28  =  485  diam. inscribed circle
 side  280  =  112 + 112 + 56
       194 + 194 + 97  =  485  diameter inscribed circle
 side  280  =  194 +  71 + 15
       336 + 123 + 26  =  485  diameter inscribed circle
```

If the side of a regular hexagon measures 10 royal cubits or 70 palms or 280 fingers, the diameter of the inscribed circle measures practically 485 fingers or 17 cubits 2 palms 1 finger (mistake half a millimeter).

The edge of a cube measures 10 royal cubits. Calculate the cubic diagonal. It measures again 485 fingers or 17 cubits 2 palms 1 finger (with a tiny error of only half a millimeter).

Double square

Yet another number pattern:

```
 1        1        5
   2        6       10
   1        3        5
     4        8       20
     2        4       10
     1        2        5
       3        7       15
         10       22       50
          5       11       25
           16       36       80
            8       18       40
            4        9       20
             13       29       65
               42       94      210
               21       47      105
                 68      152      340
                 34       76      170
                 17       38       85
                   55      123      275
                    178      398      890
                     89      199      445
                      288      644     1440
                      144      322      720
                       72      161      360
```

If a double square measures 4 by 8 royal cubits or 34 by 68 royal cubits or 72 by 144 royal cubits, the diagonals measure about 9 or 76 or 161 royal cubits, with an increasing accuracy. - Let a rectangle measure 10 by 20 royal cubits or 280 by 560 fingers. Calculate the diagonal. Proceed as follows:

```
 short side  280 = 144 +  89 + 34 + 13
                   322 + 199 + 76 + 29 = 626  diagonal
 short side  280 = 199 +  76 +  5
                   445 + 170 + 11 = 626  diagonal
```

If a double square measures 10 by 20 royal cubits or 70 by 140 palms or 280 by 560 fingers, the diagonal measures practically 626 fingers or 22 cubits 2 palms 2 fingers (error 2 millimeters).

Doubling the volume of a cube

```
 1       1       1       2
   2       2       3       4
     4       5       7       8
       9      12      15      18
       3       4       5       6
 3       4       5       6
   7       9      11      14
    16      20      25      32
      36      45      57      72
      12      15      19      36
12      15      19      24
  27      34      43      54
    61      77      97     122
     138     174     219     276
      46      58      73      92
46      58      73      92
 104     131     165     208
   235     296     373     470
     531     669     843    1062
     177     223     281     354
177     223     281     354
 400     504     635     800   and so on
```

A cube measures 400 by 400 by 400 units. Double the volume and the cube will measure 504 by 504 by 504 units. The numbers 504 and 400 provide an excellent approximate value for the cube root of 2: 504/400 = 63/50.

Calculating the circle

Imagine a grid measuring 10 by 10 royal cubits:

```
. . . . . d . . . . .
. . e . . . . . c . .
. f . . . . . . . b .
. . . . . . . . . . .
. . . . . . . . . . .
g . . . . + . . . . a
. . . . . . . . . . .
. . . . . . . . . . .
. h . . . . . . . l .
. . i . . . . . k . .
. . . . . j . . . . .
```

The side of the square measures 10 royal cubits or 70 palms or 280 fingers. The diagonal measures practically 99 palms. The points a b c d e f g h i j k l mark a circle whose radius measures 5 royal cubits or 35 palms or 140 fingers. The eight short arcs measure about 40 fingers each, the four longer arcs measure practically 90 fingers each, giving a circumference of about 880 fingers or 220 palms and yielding a very fine approximate value for pi, namely 22/7 or 3 1/7.

Imagine a grid which measures 10 by 10, 50 by 50, 250 by 250, 1250 by 1250 ... ever smaller units. A circle inscribed in it will pass the 4 ends of the axes, furthermore 8, 16, 24, 32 ... inner points of the grid. Their distances from the axes and from the center of the grid are defined by the following triples, beginning with the 'Sacred Triangle' 3-4-5:

```
3-4-5  or  15-20-25  or  75-100-125  or  375-500-625  ...
      7-24-25  or  35-120-125  or  175-600-625  ...
            44-117-125  or  220-585-625  ...
                   336-527-625  ...
```

If you know a triple a-b-c and wish to know the next, you may calculate according to the following terms:

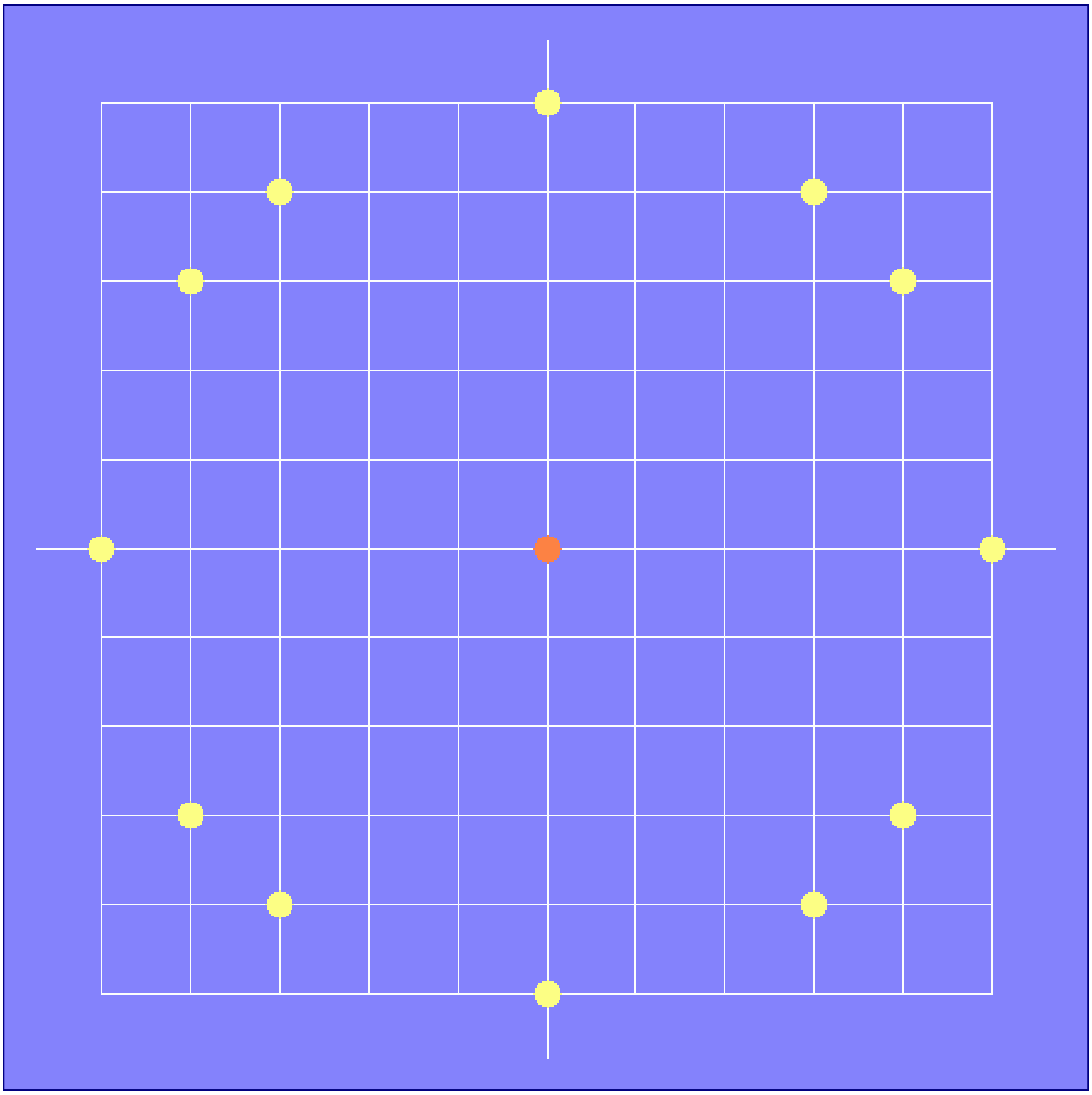
```
4a plus/minus 3b   4b plus/minus 3a   5c
```

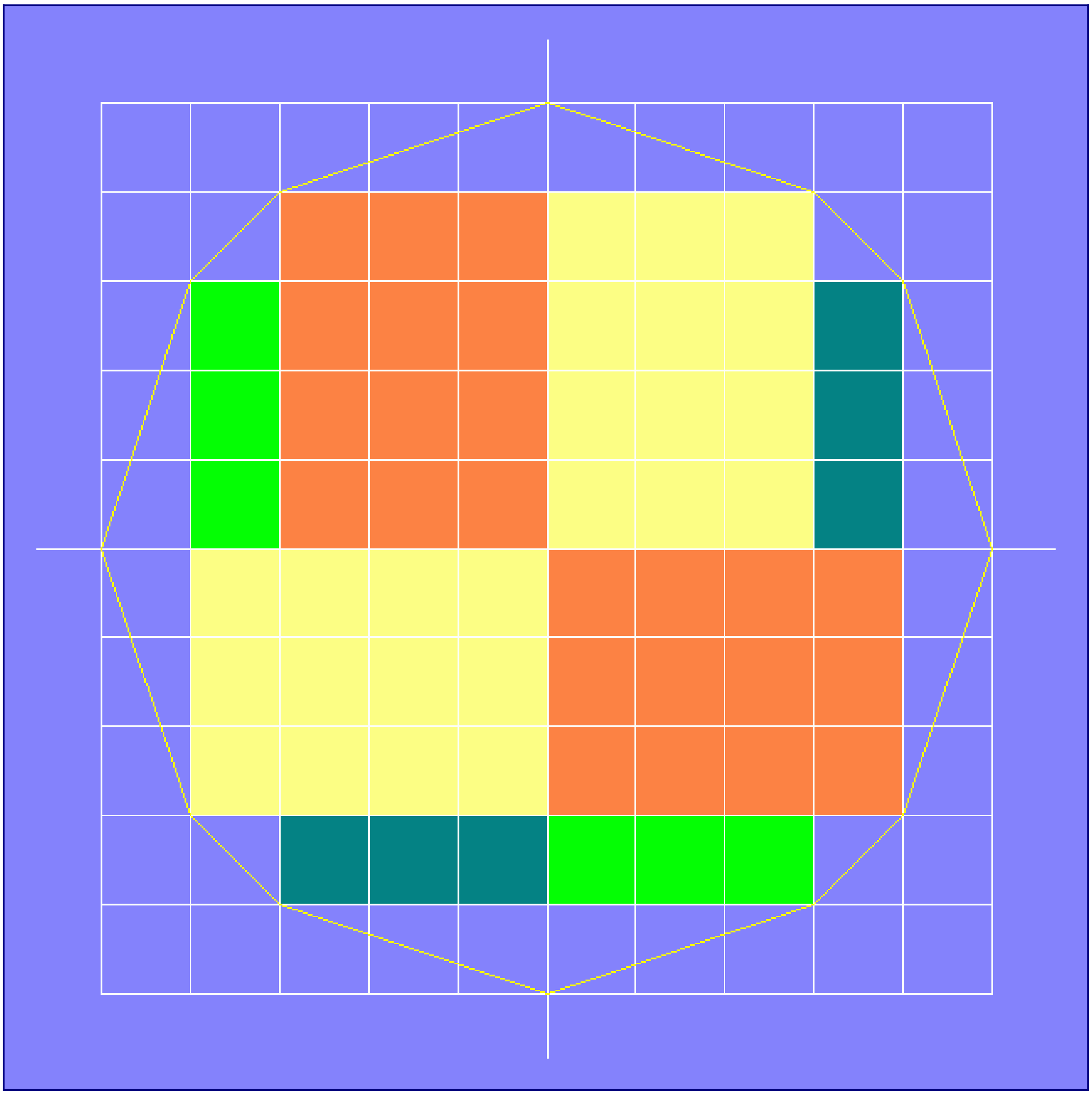
Use the positive results ending on 1, 2, 3, 4, 6, 8 or 9 (neither on zero nor on five).

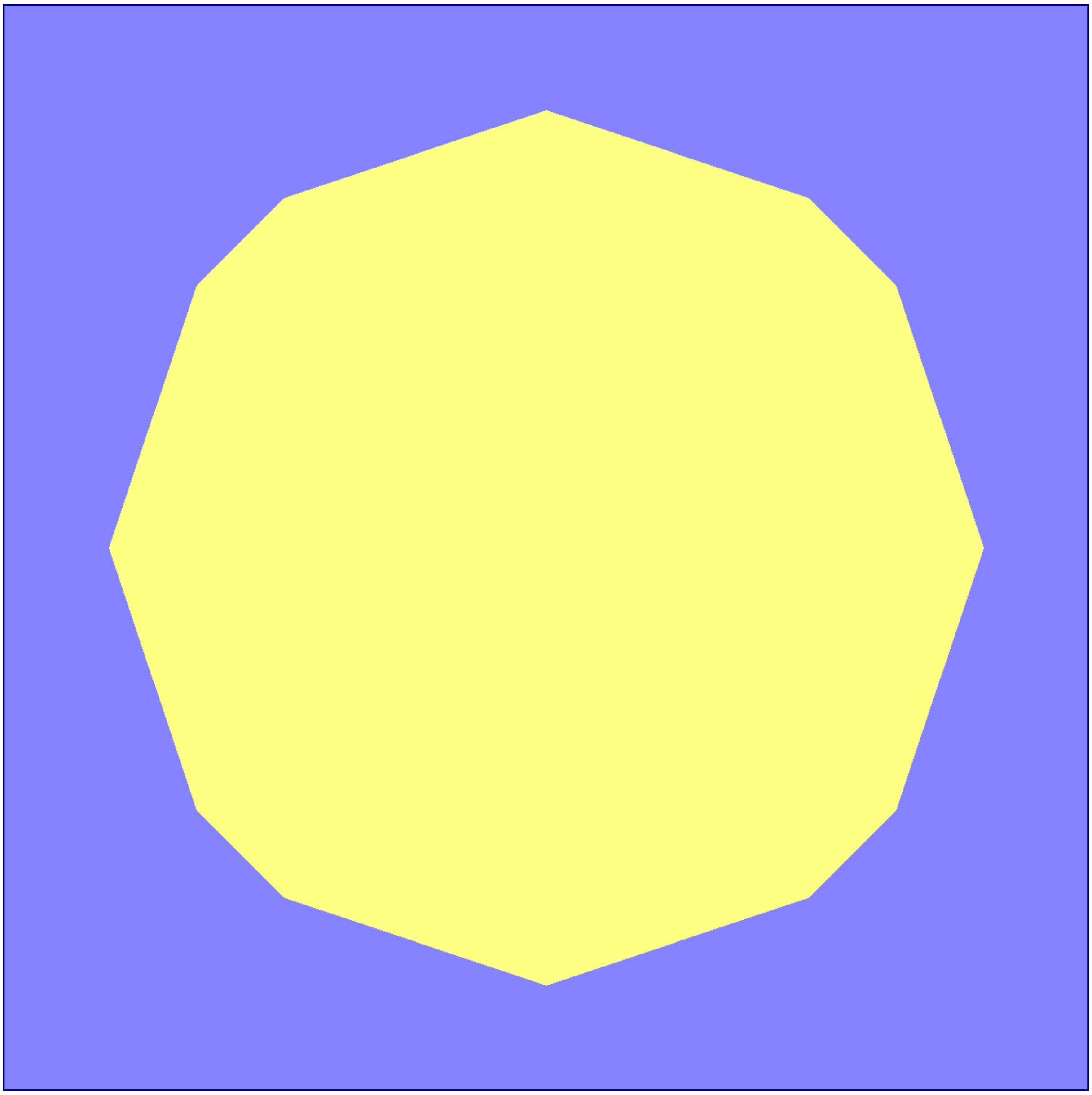
Combine the 12, 20, 28, 36 ... points by straight lines and you will obtain a sequence of irregular polygons. Their side lengths are whole number multiples of the square roots of 2 or 5 or 2x5. The square roots of 2 and 5 can be approximated by means of two of the number patterns above.

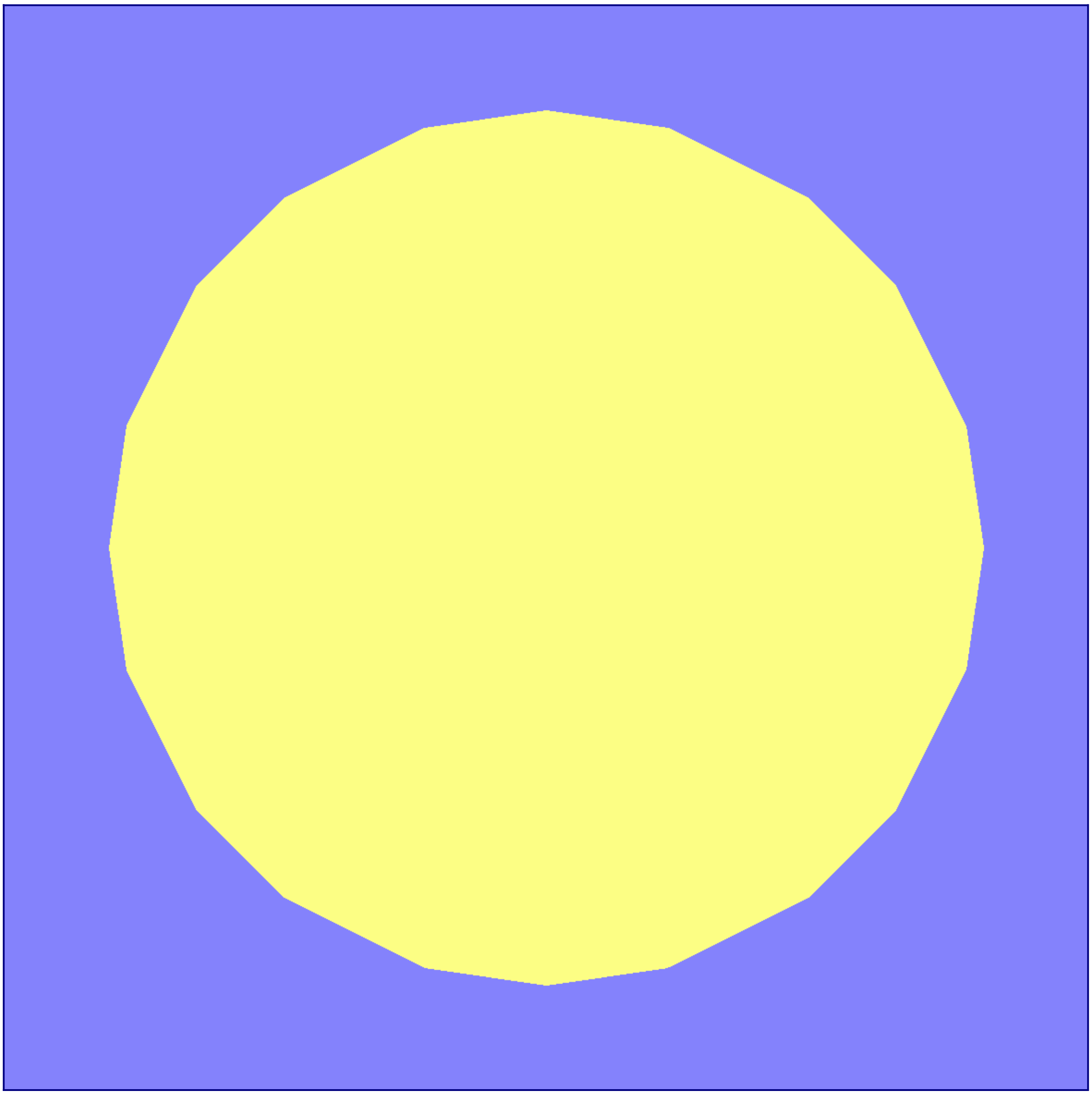
The sides of the irregular polygons are slightly shorter than the respective arcs. We may hope to counterbalance this by choosing values for the square roots of 2 and 5 that are slightly bigger than the actual numbers. Calculate the first polygon by means of the ratios 10/7 and 9/4 and you will find the ratio 22/7 for pi. Calculate the second polygon by means of the ratios 17/12 and again 9/4 and you will find 157/50 for pi. The average is about 311/99. These values allow to draw up a number sequence providing many more fine values. Write 3 above 1 and add continuously 22 above 7:

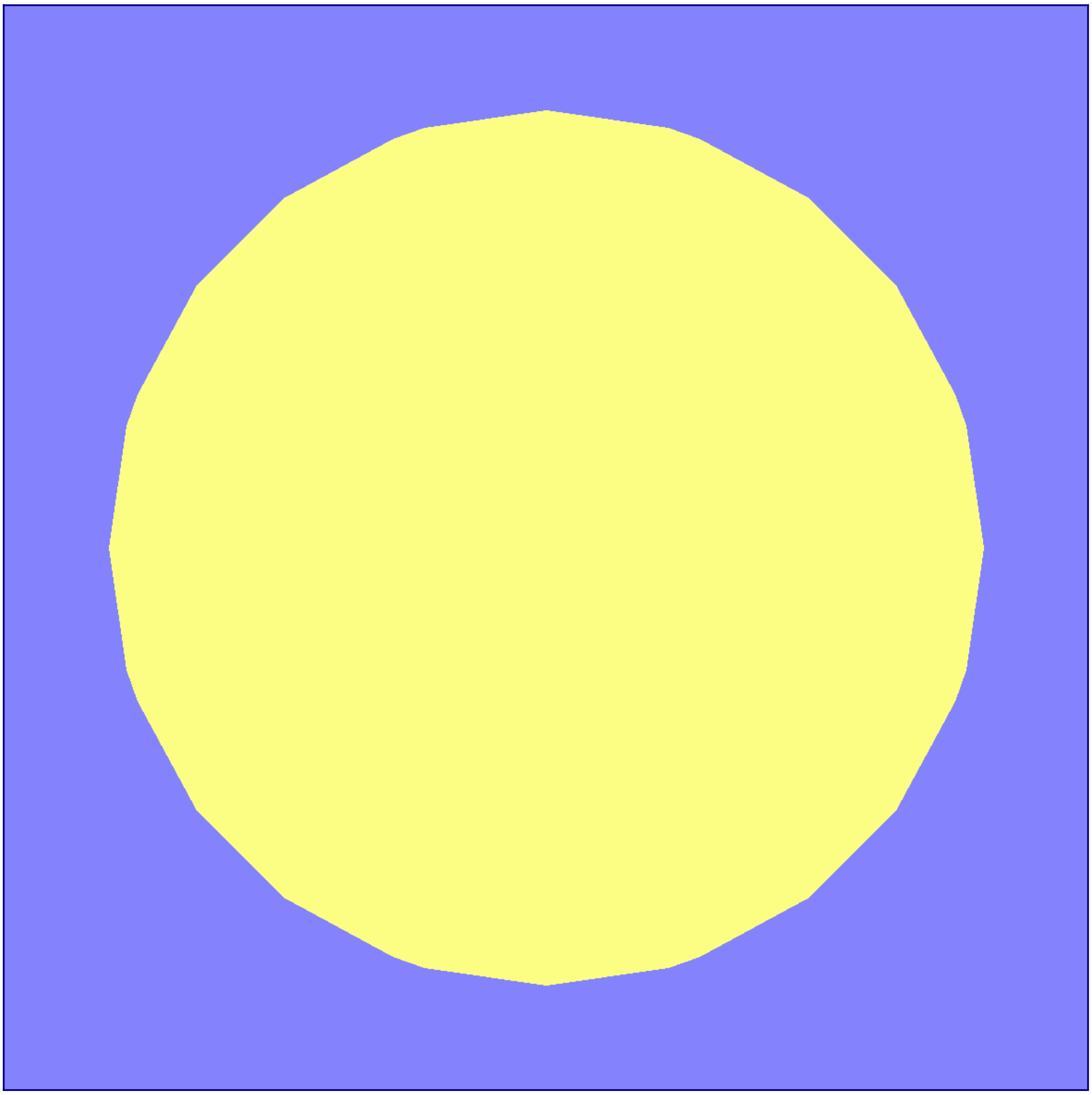
```
3  (plus 22)  25  47  ...  157  ...  311  333  355  377
1  (plus  7)   8  15  ...   50  ...   99  106  113  120
```

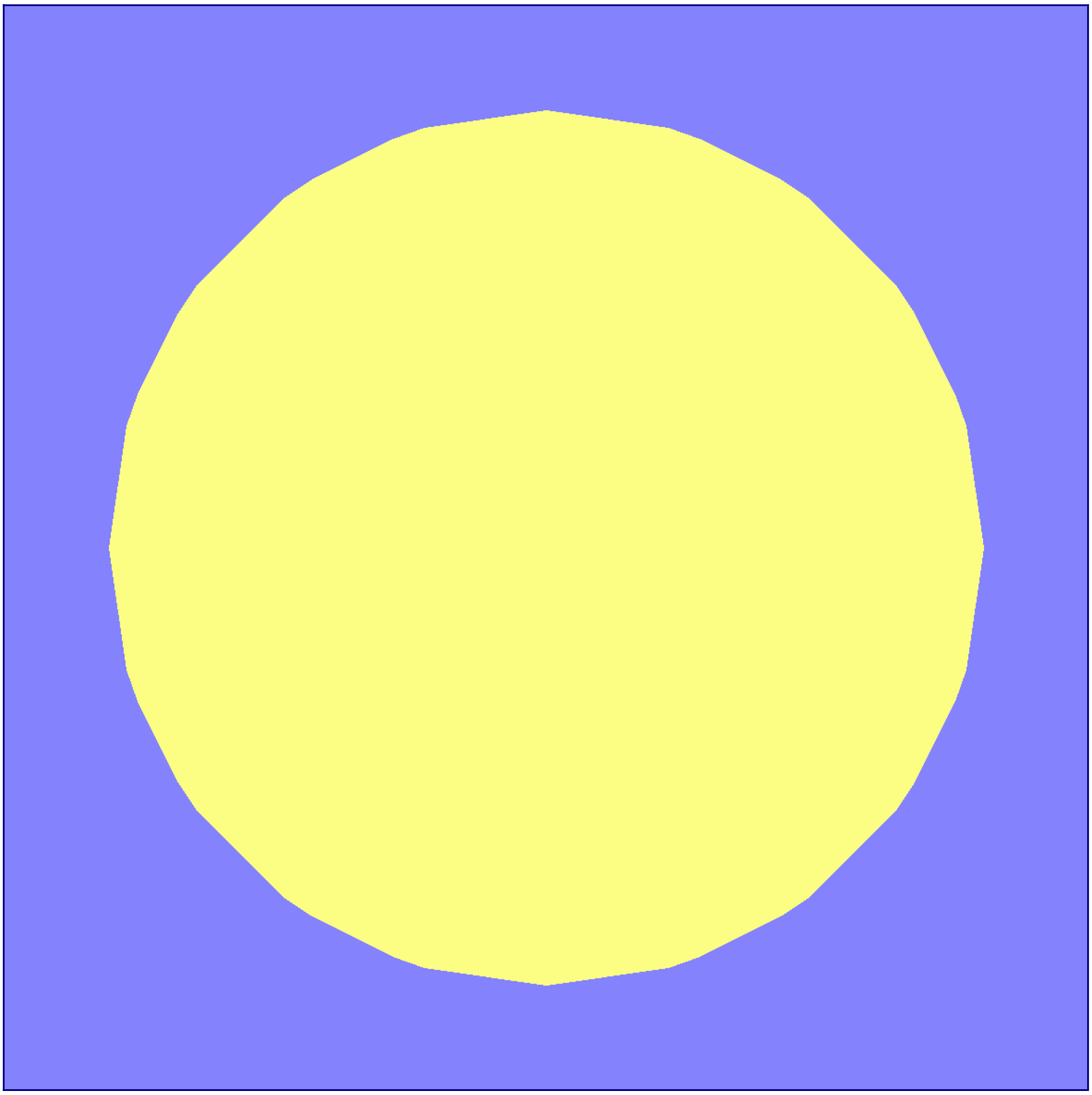


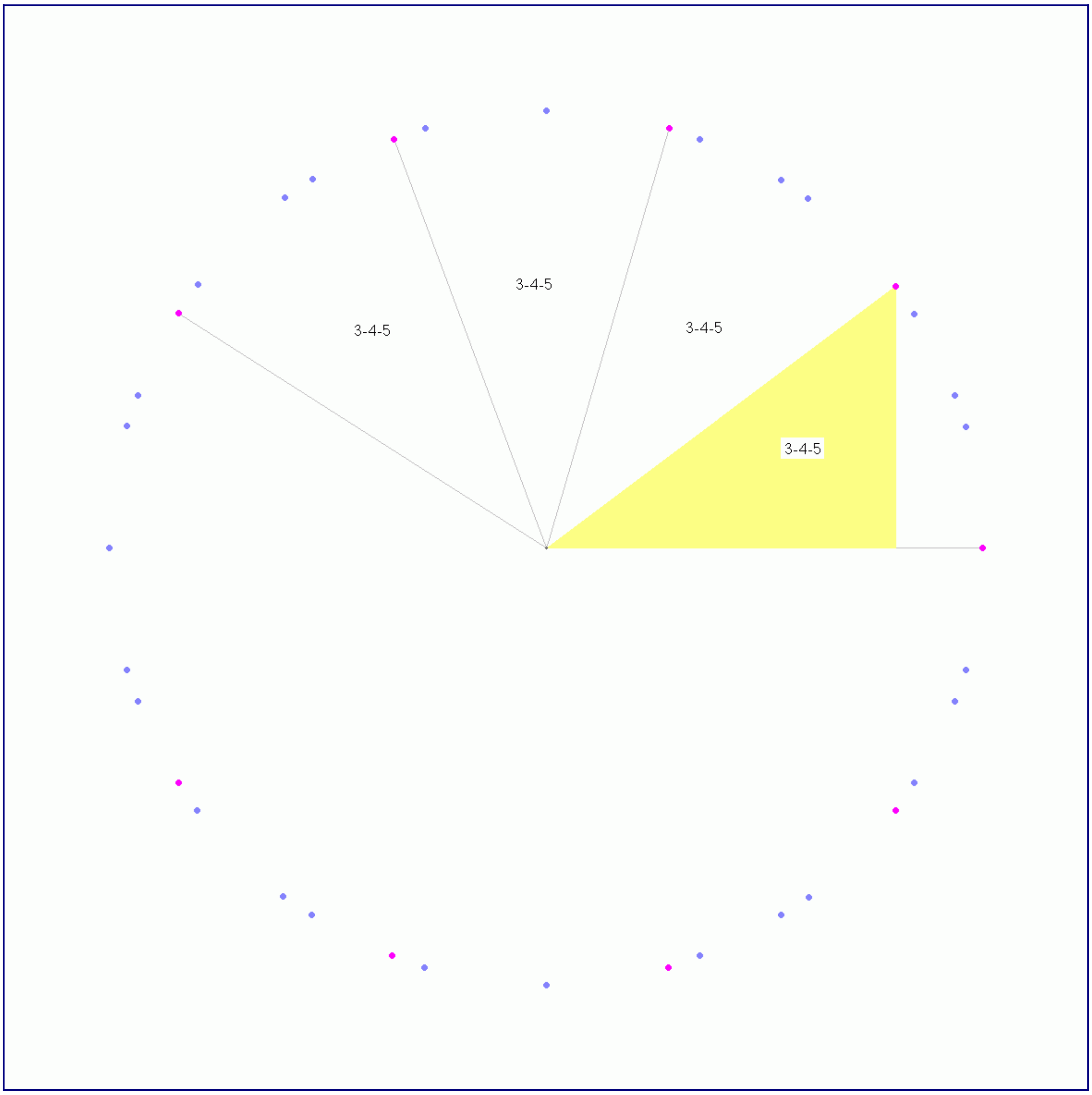


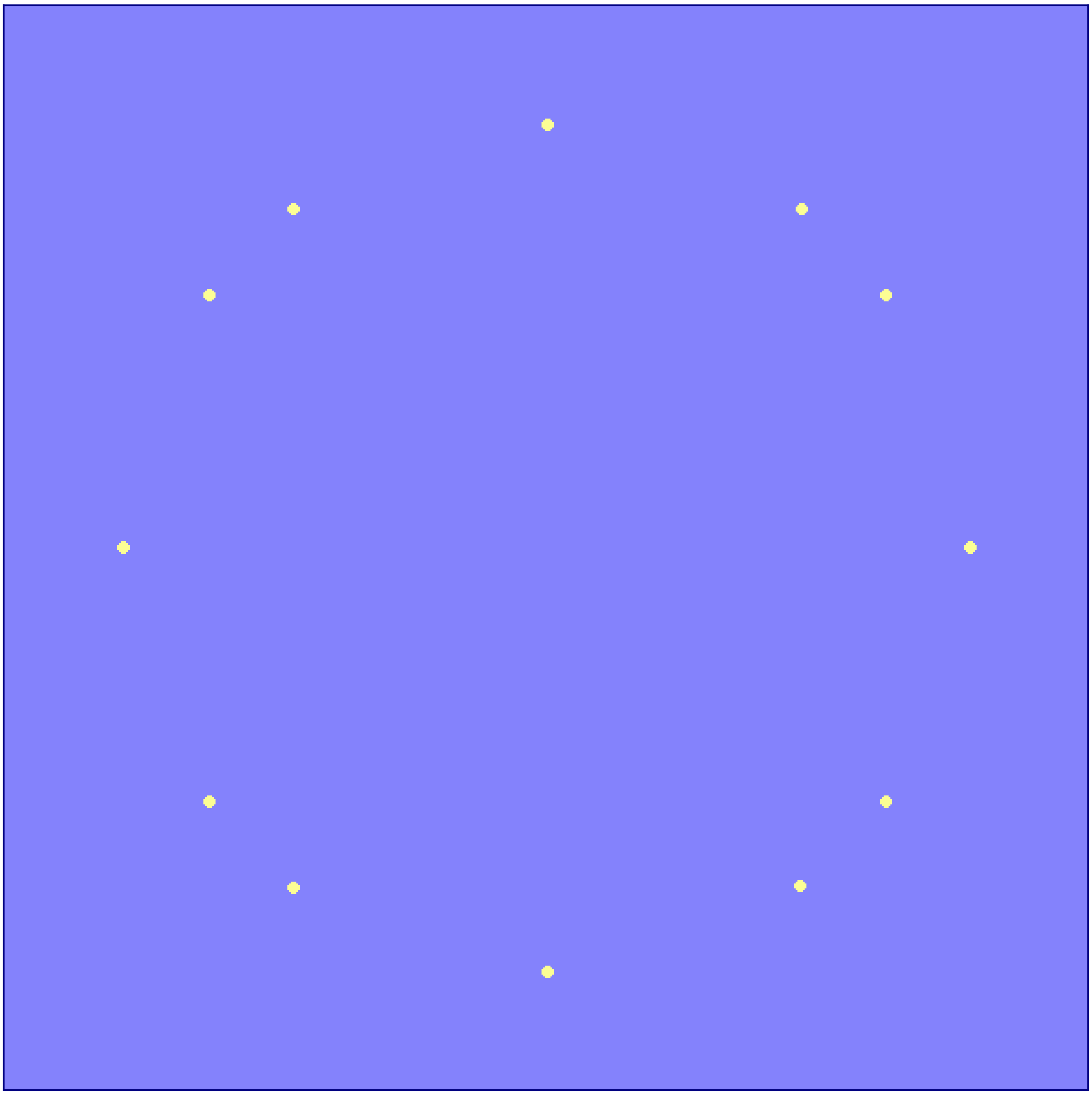


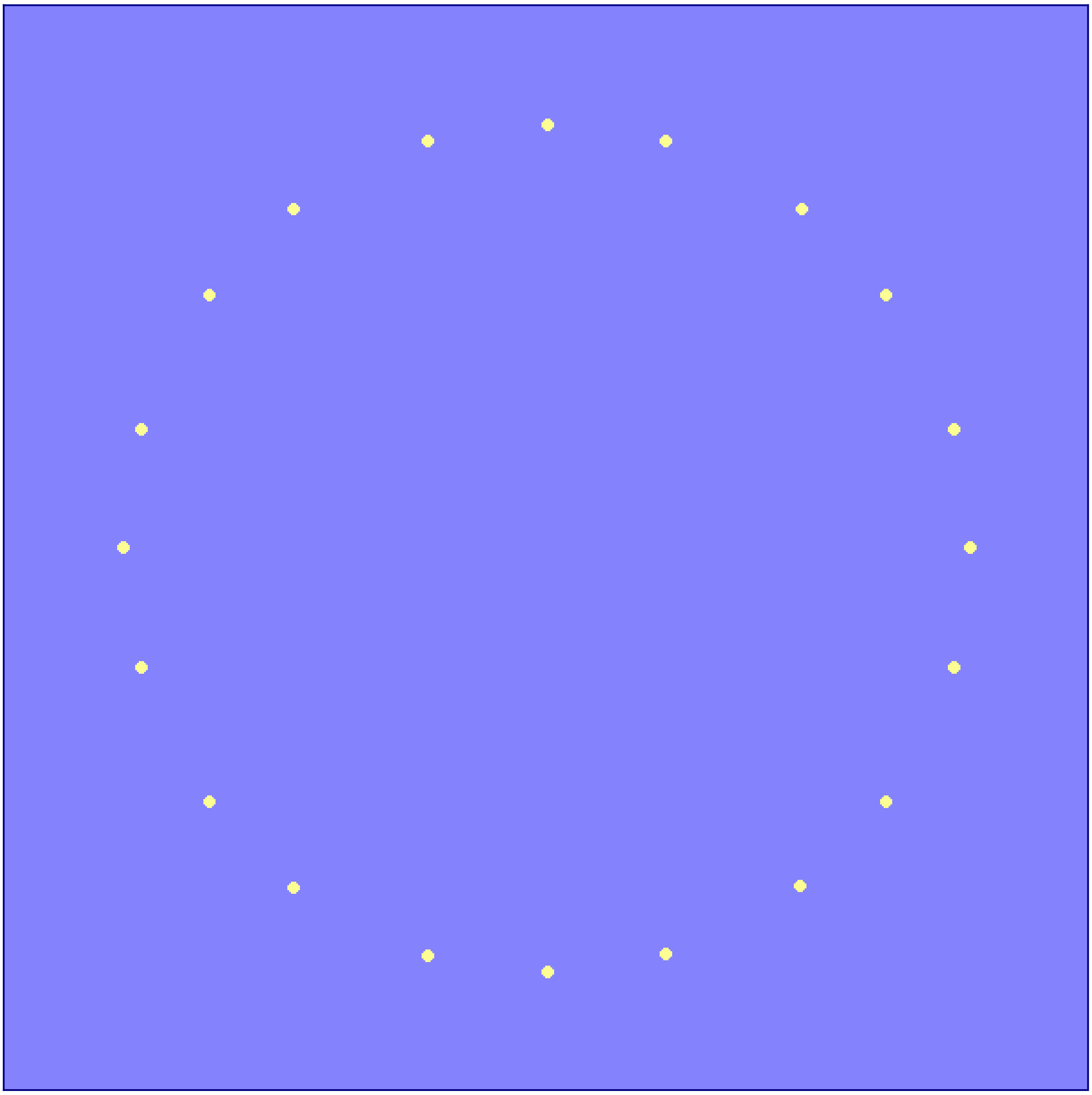


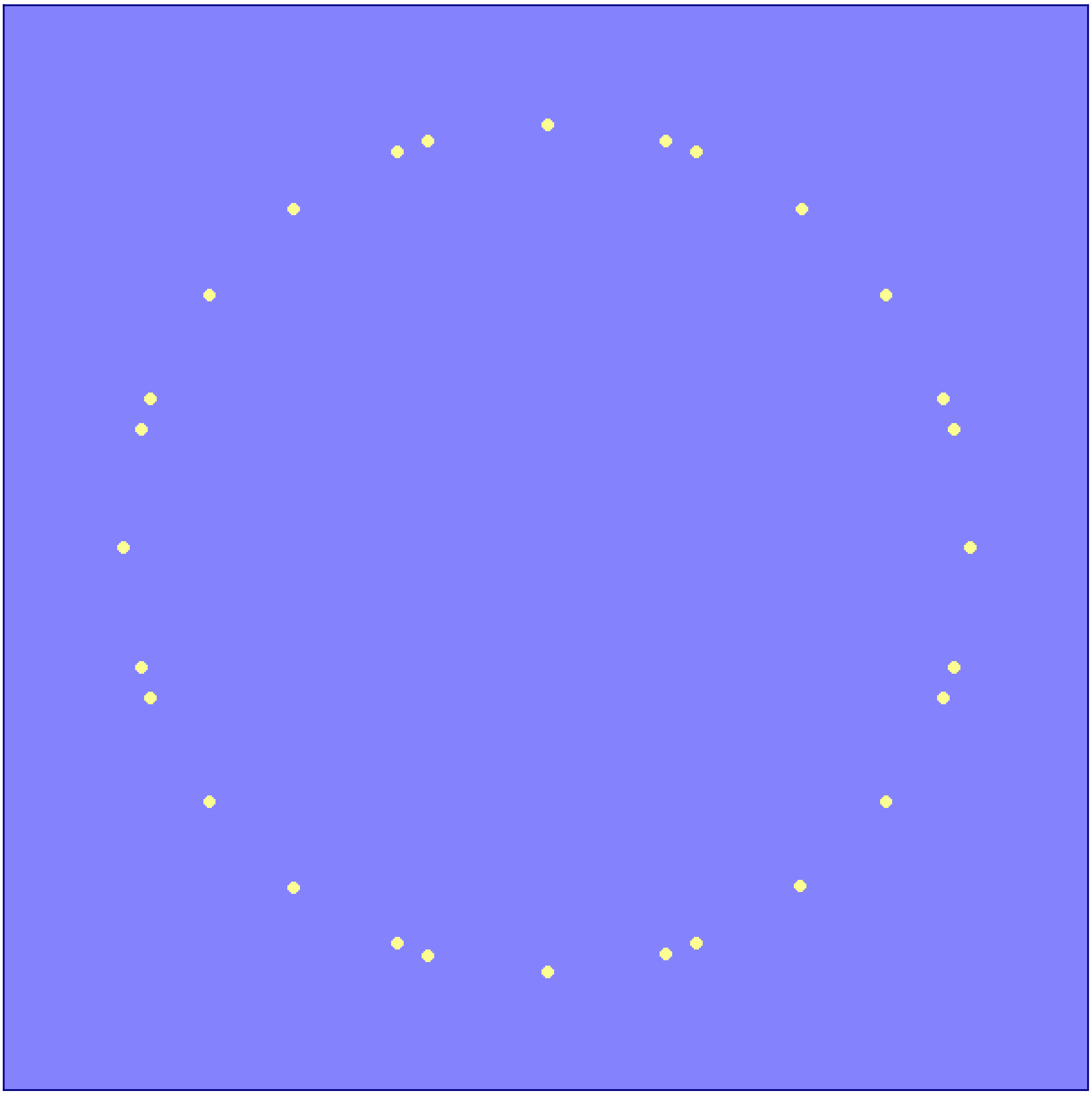


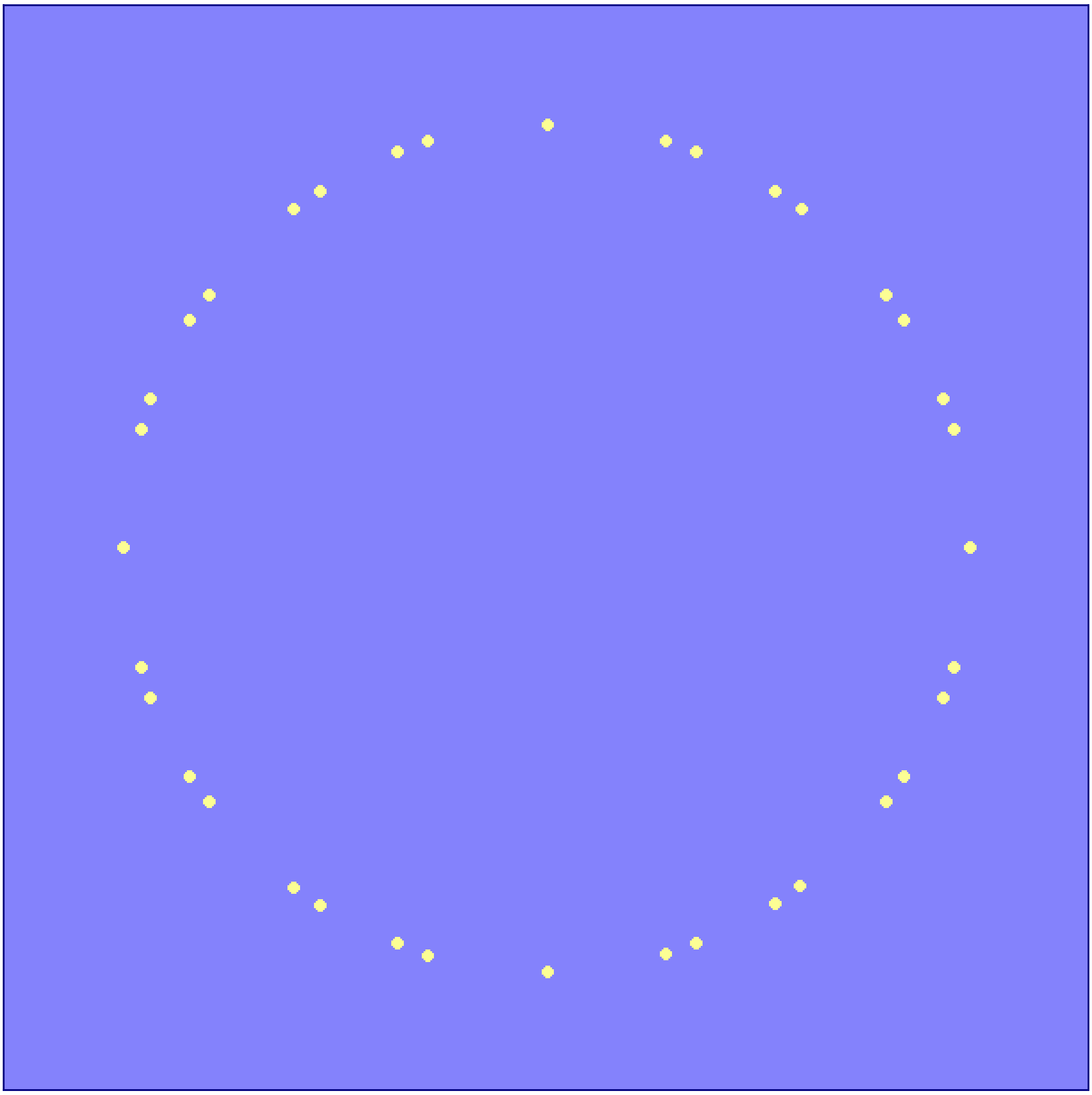



3-4-5
3-4-5
3-4-5
3-4-5









By the way: the above triples can also be found by means of a number column. Begin with 1 and 1, use a factor of minus 4, and consider every second line:

```
1       1      -4
  2      -3      -8
   -1     -11      4
     -12      -7      48
       -19      41     76
          22      117    -88   and so on
x =  -3      y =  -8/2 =  -4      r =   5
x =  -7      y =  48/2 =  24      r =  25
x = 117      y = -88/2 = -44      r = 125   and so on
```

Cumbersome? No, a clever tool

Many historians of mathematics believe that unit fractions are cumbersome. Are they? I tried to work with them and much to my surprise found an easy way to handle those funny numbers: round all results, and the mistakes will even out in the long run, allowing you to work with whole numbers only, and yielding fine results all the same.

In my first number column are found the numbers 70 and 99, yielding the value 99/70 for the square root of 2. Now let me transform this ratio into a pair of unit fraction series:

```
99/70  =  1 + 1/5 + 1/7  + 1/14  or simply  1 '5 '7 '14
99/70  =  1 + 1/3 + 1/15 + 1/70  or simply  1 '3 '15 '70
```

Let the side of a square measure 360 royal cubits or 2520 palms. If you wish to calculate the diagonal you may multiply the side by one of the above series and round all the numbers:

```
360 royal cubits  x  1 '5 '7 '14
360 x 1    =  360
360 x '5   =   72
360 x '7   =   51 (rounded)
360 x '14  =   26 (rounded)
sum           509
side          360 royal cubits
diagonal      509 royal cubits  (mistake about 6 centimeters)
2520 palms  x  1 '5 '7 '14
2520 x 1    =  2520
2520 x '5   =   504
2520 x '7   =   360
sum            3564
side           2520 palms
diagonal       3564 palms  (mistake about 14 millimeters)
```

Khafre's pyramid at Giza was originally 274 royal cubits tall, while the base measured 411 royal cubits and the slope 324 '2 royal cubits, according to the Sacred Triangle 3-4-5. How long was the diagonal of the base?

```
411 x 1    =  411
```

```
 411 x '5   =   82 (rounded)
 411 x '7   =   59 (rounded)
 411 x '14  =   29 (rounded)
 sum           581
 side          411 royal cubits
 diagonal      581 royal cubits  (mistake about 13 centimeters)
411 royal cubits equal 2877 palms:
 2877 x 1    =  2877
 2877 x '5   =   575 (rounded)
 2877 x '7   =   411
 2877 x '14  =   205 '2  ??
```

Here we have a rounding problem. 2877 divided by 14 equals 205 plus '2. Should we round up to 206? or down to 205? No, we shall solve the problem by using the alternative series 1 '3 '15 '70:

```
 2877 x 1    =  2877
 2877 x '3   =   959
 2877 x '15  =   192 (rounded)
 2877 x '70  =    41 (rounded)
 sum            4069
 side           2877 palms
 diagonal       4069 palms  (mistake about 23 millimeters)
411 royal cubits equal 2877 palms or 11508 fingers:
 11508 x 1    =  11508
 11508 x '3   =   3836
 11508 x '15  =    767 (rounded)
 11508 x '70  =    164 (rounded)
 sum             16275
 side       11508 fingers or 411 royal cubits or
 diagonal   16275 fingers or 581 cubits 7 fingers
 (mistake 4 millimeters)
```

The base length of Khafre's pyramid measured 411 royal cubits, while the diagonal of the base measured 581 cubits 7 fingers (with a tiny mistake of only four millimeters).

Multiplying unit fraction series

How much is 184 times 2 '6 '7 times 5 '3 '5 ?

```
 184 times 2 '6 '7  equals   368   31   26  =   425
 425 times 5 '3 '5  equals  2125  142   85  =  2352 *
 184 times 5 '3 '5  equals   920   61   37  =  1018
1018 times 2 '6 '7  equals  2036  170  145  =  2351 *
 average result  2351 '2  (mistake less than '10)
```

How much is 317 times 3 '3 '11 times 4 '5 '13?

```
 317 times 3 '3 '11  equals   951  106   29  =  1086
1086 times 4 '5 '13  equals  4344  217   84  =  4645
 317 times 4 '5 '13  equals  1268   63   24  =  1355
1355 times 3 '3 '11  equals  4065  452  123  =  4640
 average result  4642 '2  (mistake less than '30)
```

How much is 5 '3 '7 '11 by 5 '3 '7 '11? This time we multiply the product by a factor of 100 and then divide the result by 100 again:

```
 100 times 5 '3 '7 '11  equals   500   33  14   9  =   556
 556 times 5 '3 '7 '11  equals  2780  185  79  51  =  3095
 3095 / 100 = practically 31  (mistake less than '135)
```

A calculation of interest

Vizier Milo (a former incarnation of Milo Gardner, whom I shall mention later :-) saved 68,954 Egyptian Dollars and brings the money to the First City Bank of Amarna, which offers interest at '101 '202 '303 '606 (2/101, a little more than 2 percent). How will Milo's fortune rise in the coming years?

```
Year 1   fortune  68,954   interest  683 341 228 114  =  1,366
year 2   fortune  70,320   interest  696 348 232 116  =  1,392
year 3   fortune  71,712   interest  710 355 237 118  =  1'420
year 4   fortune  73,132   interest  724 362 241 121  =  1,448
year 5   fortune  74,580   interest  738 369 246 123  =  1,476
year 6   fortune  76,056   interest  753 377 251 126  =  1,507
year 7   fortune  77,563   interest  768 384 256 128  =  1,536
year 8   fortune  79,099   interest  783 392 261 131  =  1,567
year 9   fortune  80,666   interest  799 399 266 133  =  1,597
year 10  fortune  82,263   interest  814 407 271 136  =  1,628
year 11  fortune  83,891   interest  831 415 277 138  =  1,661
year 12  fortune  85,552   interest  847 424 282 141  =  1,694
year 13  fortune  87,246   interest  864 432 288 144  =  1,728
year 14  fortune  88,974   interest  881 440 294 147  =  1,762
YEAR 15  FORTUNE  90,736 DOLLARS
```

And the exact value?

68954 x (1 + 2/101) exp 14 = 68954 x 1.3158971... = 90736.365...

Although we have rounded all numbers, the margin of error is not even 40 cents. And if we begin with 6,895,400 cents instead of 68,954 dollars, the mistake would be less than 4 (four) cents.

Playing with beans

```
   o o o       o o o o
  o o o o       o o o o      o o o o o o
 o o o o o      o o o o      o o o o o o      oooooooooooo
 3 + 4 + 5  =   3 x 4   =     2 x 6     =     1 x 12
 oooooooooooo      12         or    1
 oooooo oooooo      6+6       or    1/2 + 1/2
```

```
 oooooo oo oooo      6+2+4     or    1/2 + 1/6 + 1/3
 oooooo oo o ooo     6+2+1+3   or    1/2 + 1/6 + 1/12 + 1/4
```

1/1 = '1   1/2 = '2   1/3 = '3   1/4 = '4   1/6 = '6   1/12 = '12

```
  1 = '1                  1 = '1
  1 = '2 '2               1 = '1x2 '2
  1 = '2 '6 '3            1 = '1x2 '2x3 '3
  1 = '2 '6 '12 '4        1 = '1x2 '2x3 '3x4 '4
```

RHIND MATHEMATICAL PAPYRUS, duplations and conversions

The famous Rhind Mathematical Papyrus was written around 1650 BC, and represents a copy of a lost scroll dated around 1850 BC. At the begin of the RMP are found divisions of 2 by the odd numbers from 5 up to 101. Examples:

```
 "5 = '3 '15       (2/5 = 1/3 + 1/15)
 "7 = '4 '28       (2/7 = 1/4 + 1/28)
 "9 = '6 '18       (2/9 = 1/6 + 1/18)
"11 = '6 '66
"13 = '8 '52 '104
"19 = '12 '76 '114
"35 = '30 '42
"43 = '42 '86 '129 '301
"67 = '40 '268 '670
"91 = '70 '130
"95 = '60 '380 '570
"99 = '66 '198
"101 = '101 '202 '303 '606
```

The calculations are carried out as follows. If you wish to divide 2 by any number a, note the numbers 1 and a. Divide them by handy numbers until you get a number b that is smaller than 2. Now subtract number b from 2 and complete your series. Examples:

```
 1         9  (number a)
 '3        3
 '6        1 '2  (number b)
 2 minus 1'2 equals '2
 '18       '2
 "9 = '6 '18  (since 1'2 '2 equals 2)
 1         19
 '3         6 '3       (divided by 3)
 '6         3 '6       (divided by 2)
 '12        1 '2 '12   (divided by 2)
 2 minus 1'2'12 equals ??
24 minus 12+6+1 equals 5 or 3 + 2     (multiplied by 12)
 2 minus 1'2'12 equals      '4  '6   (divided by 12)
 '76        '4
 '114       '6
```

```
"19 = '12 '76 '114  (since 1'2'12 '4 '6 equals 2)
1           35
'5           7
'15          2 '3
'30          1 '6
2 minus 1'6 equals '2 '3
 35 divided by 2+3 equals 7 / remainder '2x3x7 equals '42
"35 = '30 '42
 1        43
'2         21 '2   (divided by 2)
'6          7 '6   (divided by 3)
'42         1 '42  (divided by 7)
2 minus 1'42 equals '2 '3 '7
'86         '2
'129        '3
'301        '7
"43 = '42 '86 '129 '301  (since 1'42 '2 '3 '7 equals 2)
1          91
'7         13
'14         6 '2
'70         1 '5 '10
2 minus 1'5'10 equals '2 '5
 91 divided by 2+5 equals 13 / remainder '2x5x13 or '130
"91 = '70 '130
1         93
'31        3
'62        1'2
2 minus 1'2 equals '2
'186       '2
"93 = '62 '186  (since 1'2 '2 equals 2)
1         101
'101        1
2 minus 1 equals '2 '3 '6
'202   '2      '303   '3      '606   '6
"101 = '101 '202 '303 '606  (since 1 '2 '3 '6 equals 2)
```

Down under algebra

Beginners may carry out all divisions from 2/5 to 2/101 and in so doing learn how to work with unit fraction series. Advanced learners may go a step further and look out for number patterns providing the same conversions:

```
 1  =  '2 '1x2  =  '2 '2
'2  =  '3 '2x3  =  '3 '6
'3  =  '4 '3x4  =  '4 '12
'4  =  '5 '4x5  =  '5 '20
'5  =  '6 '5x6  =  '6 '30
'6  =  '7 '6x7  =  '7 '42
'7  =  '8 '7x8  =  '8 '56      general form: 'a = 'a+1 'aa+a
```

```
 1  =  '2 '2  --------  "1 = '1 '1
'2  =  '3 '6
'3  =  '4 '12  -------  "3 = '2 '6
'4  =  '5 '20
'5  =  '6 '30  -------  "5 = '3 '15 (RMP)
'6  =  '7 '42
'7  =  '8 '56  -------  "7 = '4 '28
'8  =  '9 '72
'9  =  '10 '90  ------  "9 = '5 '45 (RMP)
'10  =  '11 '110
'11  =  '12 '132  ----  "11 = '6 '66 (RMP)
..............................................
```

The first number pattern generates a pair of remarkable series:

```
1 = '2 '2
   '2 = '6 '3
       '3 = '12 '4
           '4 = '20 '5
               '5 = '30 '6
                   '6 = '42  ...
1 = '2      '6      '12      '20      '30      '42  ...
1 = '1x2    '2x3    '3x4     '4x5     '5x6     '6x7 ...
1 = '1
1 = '1x2 '2
1 = '1x2 '2x3 '3
1 = '1x2 '2x3 '3x4 '4
1 = '1x2 '2x3 '3x4 '4x5 '5
1 = '1x2 '2x3 '3x4 '4x5 '5x6 '6
1 = '1x2 '2x3 '3x4 '4x5 '5x6 '6x7 '7
.......................................
1 = '2 '2
   '2 = '3 '6
       '6 = '7 '42
           '42 = '43 1806
1 = '1
1 = '2 '2
1 = '2 '3 '6
1 = '2 '3 '7 '42
1 = '2 '3 '7 '43 '1806
1 = '2 '3 '7 '43 '1807 '3263443 ...
```

The equation "3 = '2 '6 can be used for many simple conversions:

```
 "9 equals '6  '18   (RMP)
"15 equals '10 '30   (RMP)
"21 equals '14 '42   (RMP)
..........................
"87 equals '58 '174  (RMP)
```

```
"93 equals '62 '186  (RMP)
"99 equals '66 '198  (RMP)
```

The principle of the first number pattern may be expanded as follows:

```
 1 = '2 '2   = '3 '3 '3    = '4 '4 '4 '4      ...
'2 = '3 '6   = '4 '8 '8    = '5 '10 '10 '10  ...
'3 = '4 '12  = '5 '15 '15  = '6 '18 '18 '18  ...
'4 = '5 '20  = '6 '24 '24  = '7 '28 '28 '28  ...
'5 = '6 '30  = '7 '35 '35  = '8 '40 '40 '40  ...
.........................................
```

Modifying the third column:

```
 1  =  '4  '4   '2  -----------  "1 = '2 '2 '1
'2  =  '5  '10  '5
'3  =  '6  '18  '9
'4  =  '7  '28  '14
'5  =  '8  '40  '20  ----------  "5 = '4 '20 '10
'6  =  '9  '54  '27
'7  =  '10 '70  '35
'8  =  '11 '88  '44
'9  =  '12 '108 '54  ----------  "9 = '6 '54 '27
'10  =  '13 '130 '65
'11  =  '14 '154 '77
'12  =  '15 '180 '90
'13  =  '16 '208 '104  --------  "13 = '8 '104 '52 (RMP)
```

A more demanding general pattern:

```
 "1  equals  '1  plus  '1x1 of 1     (2/1 = 1/1 + 1/1x1)
 "3  equals  '3  plus  '3x3 of 3     (2/3 = 1/3 + 3/3x3)  3=2+1
       '2  plus  '3x2 of 1     (2/3 = 1/2 + 1/2x3)
 "5  equals  '5  plus  '5x5 of 5     (2/5 = 1/5 + 5/5x5)  5=3+2
       '4  plus  '5x4 of 3     (2/5 = 1/4 + 3/4x5)  3=2+1
       '3  plus  '5x3 of 1     (2/3 = 1/3 + 1/3x5)
 "7  equals  '7  plus  '7x7 of 7     (2/7 = 1/7 + 7/7x7)  7=4+3
       '6  plus  '7x6 of 5     (2/7 = 1/6 + 5/6x7)  5=3+2
       '5  plus  '7x5 of 3     (2/7 = 1/5 + 3/5x7)  3=2+1
       '4  plus  '7x4 of 1     (2/7 = 1/4 + 1/4x7)
 "9  equals  '9  plus  '9x9 of 9
       '8  plus  '9x8 of 7
       '7  plus  '9x7 of 5
       '6  plus  '9x6 of 3
       '5  plus  '9x5 of 1
 ......................................................
```

All these and many more number patterns are contained in Milo Gardner's formulas:

$$2/p - 1/a = (2a - p) / pa$$

$$n/p - 1/a = (na - p) / pa$$

It was Milo Gardner who stimulated my interest in unit fractions, back in Spring 1997. I thank him again for his many patient e-mails.

A short way to pi

The famous Egyptian Horus eye series 1 = '2 '4 '8 '16 '32 '64 (...) can be developed by means of a stairway:

```
 1 = '1
 1 = '2 '2
 1 = '2 '4 '4
 1 = '2 '4 '8 '8
 1 = '2 '4 '8 '16 '16
 1 = '2 '4 '8 '16 '32 '32
 1 = '2 '4 '8 '16 '32 '64 '64
 .................................
```

Stairway approximating 1:

```
 '2
 '2 '2x2
 '2 '2x2 '2x2x2
 '2 '2x2 '2x2x2 '2x2x2x2
 '2 '2x2 '2x2x2 '2x2x2x2 '2x2x2x2x2
 '2 '2x2 '2x2x2 '2x2x2x2 '2x2x2x2x2 '2x2x2x2x2x2
 ..............................................................
```

Resulting series:

```
 1 = '2 '2x2 '2x2x2 '2x2x2x2 '2x2x2x2x2 '2x2x2x2x2x2 ...
 1 = '2  '4    '8     '16       '32          '64      ...
```

One eye of the Horus falcon was the moon; his other eye was the sun. If there had been a second Horus eye series it might well have been this one:

```
 1 = '1
 1 = '1x2 '2
 1 = '1x2 '2x3 '3
 1 = '1x2 '2x3 '3x4 '4
 1 = '1x2 '2x3 '3x4 '4x5 '5
 1 = '1x2 '2x3 '3x4 '4x5 '5x6 '6
 1 = '1x2 '2x3 '3x4 '4x5 '5x6 '6x7 '7
 ...........................................
 '1x2
 '1x2 '2x3
 '1x2 '2x3 '3x4
 '1x2 '2x3 '3x4 '4x5
 '1x2 '2x3 '3x4 '4x5 '5x6
 '1x2 '2x3 '3x4 '4x5 '5x6 '6x7
 '1x2 '2x3 '3x4 '4x5 '5x6 '6x7 '7x8
 ....................................... stairway approximating 1
```

```
1 = '1x2 '2x3 '3x4 '4x5 '5x6 '6x7 '7x8 '8x9 '9x10 ...
1 =  '2   '6   '12  '20  '30  '42  '56  '72  '90  ...
```

The resulting series has a fascinating sub-series

```
1 = '1x2 '2x3 '3x4 '4x5 '5x6 '6x7 '7x8 '8x9 ...
  '1x2 '2x3           '5x6 '6x7           ... = pi/4
```

which can be transformed as follows

```
'1x2 '2x3 = '1x3    '5x6 '6x7 = '5x7    ...
'1x3 '5x7 '7x9 '11x13 '15x17 '19x21 '23x25 ... = pi/8
 '2 = '2
 '2 = '1x3 '6
 '2 = '1x3 '3x5 '10
 '2 = '1x3 '3x5 '5x7 '14
 '2 = '1x3 '3x5 '5x7 '7x9 '18
 ..................................
 '2 = '1x3 '3x5 '5x7 '7x9 '9x11 '11x13 '13x15 '15x17 ...
   '1x3      '5x7      '9x11        '13x15        ... = pi/8
```

or into the famous series

```
pi/4  =  1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + 1/13 - 1/15 ...
```

which was already known to the Indian mathematician Madhavan (c.1340-1425 AD), long before it was rediscovered by Gregory and Leibniz.

Duplations and right parallelepipeds

```
"1 = '1 '2 '2     quadruple  1-2-2-3
"2 = '2 '3 '6     quadruple  2-3-6-7
"3 = '3 '4 '12    quadruple  3-4-12-13
"4 = '4 '5 '20    quadruple  4-5-20-21
"5 = '5 '6 '30    quadruple  5-6-30-31
"6 = '6 '7 '42    quadruple  6-7-42-43
"7 = '7 '8 '56    quadruple  7-8-56-57
.......................................
```

As an example you may consider the quadruple 2-3-6-7: if a right parallelepiped measures 2 by 3 by 6 palms, the cubic diagonal measures exactly 7 palms or 1 royal cubit.

General form of the above quadruples: a --- a+1 --- aa+a --- aa+a+1. By choosing '2 '10 for the number a and by multiplying the resulting numbers by a factor of 25 you obtain the quadruple 15-40-24-49: if a right parallelepiped measures 15 by 40 by 24 palms, the cubic diagonal measures exactly 49 palms or 7 royal cubits.

```
1 = '1
1 = '1x2 '2
1 = '1x2 '2x3 '3
1 = '1x2 '2x3 '3x4 '4
1 = '1x2 '2x3 '3x4 '4x5 '5
1 = '1x2 '2x3 '3x4 '4x5 '5x6 '6x7   and so on
```

```
 1 = '2 '6 '12 '20 '30 '42 '56 '72 '90 '110 '132 ...
 "1 = '1x1 '1x2 '1x2    = '1 '2 '2      quadruple 1-2-2-3
 "2 = '1x2 '1x3 '2x3    = '2 '3 '6      quadruple 2-3-6-7
 "6 = '2x3 '2x5 '3x5    = '6 '10 '15    quadruple 6-10-15-19
"12 = '3x4 '3x7 '4x7    = '12 '21 '28   quadruple 12-21-28-37
"20 = '4x5 '4x9 '5x9    = '20 '36 '45   quadruple 20-36-45-61
"30 = '5x6 '5x11 '6x11  = '30 '55 '66   quadruple 30-55-66-91
.............................................................
 "3 = '1x3 '1x4 '3x4    = '3 '4 '12     quadruple 3-4-12-13
"15 = '3x5 '3x8 '5x8    = '15 '24 '40   quadruple 15-24-40-49
"35 = '5x7 '5x12 '7x12  = '35 '60 '84   quadruple 35-60-84-109
..............................................................
 "5 = '1x5 '1x6 '5x6    = '5 '6 '30     quadruple 5-6-30-31
"21 = '3x7 '3x10 '7x10  = '21 '30 '70   quadruple 21-30-70-79
.............................................................
```

General form: "ab = 'ab 'aa+ab 'ab+bb, quadruple ab -- aa+ab -- ab+bb -- aa+ab+bb
Every duplation "a = 'b 'c 'd ... constitutes a division of 2, and every division of 2 generates a right parallelepiped with an easily calculable cubic diagonal:

```
 2 : a = "a    quadruple  2 - a - "a - a "a
 2 : 1 = "1    quadruple  2 - 1 - "1 - 1 "1   or  2-1-2-3
 2 : 3 = "3    quadruple  2 - 3 - "3 - 3 "3   or  6-9-2-11
 2 : 5 = "5    quadruple  2 - 5 - "5 - 5 "5   or  10-25-2-27
 2 : 7 = "7    quadruple  2 - 7 - "7 - 7 "7   or  14-49-2-51
 ...........................................................
 2 : 1'1 = 1        2 - 1'1 - 1     - 2'1       or  2-2-1-3
 2 : 1'3 = 1'2      2 - 1'3 - 1'2   - 2'2'3     or  12-8-9-17
 2 : 1'5 = 1"3      2 - 1'5 - 1"3   - 2"3'5     or  30-18-25-43
 2 : 1'7 = 1'2'4    2 - 1'7 - 1'2'4 - 2'2'4'7   or  56-32-49-81
```

If the floor of a chamber measures 49 by 56 palms or 7 by 8 royal cubits, and if the height of the chamber measures 32 palms, the cubic diagonal will measure exactly 81 palms. The volume of the chamber measures exactly 2x2x2x2x2x2x2x2 = 256 cube cubits. If a right parallelepiped measures 4 by 8 by 8 royal cubits, the volume will again measure 256 cube cubits, and the cubic diagonal will measure exactly 12 royal cubits, according to the basic quadruple 1-2-2-3.

RHIND MATHEMATICAL PAPYRUS, problems no. 7-38

In my opinion, many problems of the Rhind Mathematical Papyrus may be read on several levels of learning and understanding:
level A) how to handle unit fractions series
level B) geometrical applications
level C) theorems

RMP 32 - a magic parallelepiped

Ahmes divides 2 by 1 1/3 1/4 and obtains 1 1/6 1/12 1/114 1/228:
  2 divided by 1 '3 '4 equals 1 '6 '12 '114 '228

By following Ahmes, the young pupils learn how to handle unit fraction series, while advanced learners may imagine a right parallelepiped of the following measurements:

height 2 units
length 1 '3 '4 units
width 1 '6 '12 '114 '228 units

How long is the cubic diagonal? Simply

```
 1 '3 '4 units  plus  1 '6 '12 '114 '228 units
```

or

```
 1 1  plus  '3 '6  plus  '4 '12  plus  '114 '228  units
```

or

```
  2           '2           '3               '76    units
```

Divide 2 by any number a and you obtain b (2:a = b). Use these numbers as measurements for a right parallelepiped. It will be a 'magic parallelepiped' with the following properties:

```
height            2 units
width or length   a units
length or width   b units
area base / top   ab = 2 square units
volume            2ab = 4 cubic units
cubic diagonal    a+b units
```

RMP 34 - an easy way to measure a granary

Let us imagine a granary in the shape of a magic parallelepiped:

```
volume  500 cubic cubits / capacity  750 khars or 15,000 hekat
inner height    2 units or 10 cubits or 70 palms or 280 fingers
length x width  2 square units or 50 square cubits or 2,450
        square palms or 39,200 square fingers
```

Here are four exact solutions, one of them provided by Ahmes' equation 1 '2 '4 times 5 '2 '7 '14 equals 10 in RMP 34:

```
inner height     10 royal cubits
inner length     10 royal cubits
inner width       5 royal cubits
cubic diagonal   15 royal cubits
inner height    280 fingers  (10 royal cubits or 2 units)
inner length    245 fingers  (1 '2 '4 units)
inner width     160 fingers  (5 '2 '7 '14 royal cubits)
cubic diagonal  405 fingers  (RMP 34)
inner height    280 fingers
inner length    224 fingers
inner width     175 fingers
cubic diagonal  399 fingers
inner height    280 fingers  or  70 palms
inner length    200 fingers  or  50 palms
inner width     196 fingers  or  49 palms
cubic diagonal  396 fingers  or  99 palms
```

If height, length and width measure 70, 50 and 49 palms, the cubic diagonal measures exactly 99 palms. Now consider a very good approximation:

```
inner height           280 fingers
inner length           198 fingers
inner width            198 fingers
diagonal  practically  396 fingers
```

Such a granary can be measured simply using a rope:

```
       height                length            width
o----------------------o---------------o---------------o
   10 royal cubits       198 fingers     198 fingers
o----------------------o-------------------------------o
    diagonal base                cubic diagonal
```

RMP 33 - a wooden container in the shape of a cube

37 divided by 1 ''3 '2 '7 equals 16 '56 '679 '776

Let me invent a task related to these numbers. A wooden container in the shape of a cube has an inner edge measuring 37 fingers, while the outer edge measures 41 fingers. How long are its diagonals?

```
1        1        2
  2        3       4
     5        7       10
       12       17       24
         29       41       58
```

A face of the cube measures 41 by 41 fingers, and the diagonal measures practically 58 fingers.

```
1       1        3
  2       4        6
  1       2        3
    3       5        9
      8      14       24
      4       7       12
        11      19       33
          30      52       90
          15      26       45
            41      71      123
              112     194     336
               56      97     168
```

The cube measures 41 by 41 by 41 fingers, the cubic diagonal measures practically 71 fingers. The cavity measures 37 by 37 by 37 fingers. How long is the cubic diagonal? Use the numbers 97 and 168. Multiply 37 fingers by 168/97 and you obtain the diagonal in fingers. Divide 37 fingers by 97/42 = 1 ''3 '2 '7 and you obtain the diagonal in palms. Ahmes found the number 16 '56 '679 '776, hence the diagonal measures practically 16 '56 '679 '776 palms or about 16 palms. And the volume? A surprise: it measures practically 1 ''3 '2 '7 cubic cubits:

RMP 38 - transforming a square hekat into a cylinder

Let me begin with the subdivision of the royal cubit rod of Amenhotep I:
1 royal cubit (52.5 cm) = 7 palms (7.5 cm) = 28 fingers (1.875 cm) = 56 Re marks = 84 Shu marks = 112 Tefnut marks = 140 Geb marks = 168 Nut marks = 196 Osiris marks= 224 Isis marks = 252 Seth marks = 280 Nephtys marks = 308 Horus marks = 336 Imsety marks = 364 Hapy marks = 392 Duamutf marks = 420 Qhebsenuf marks = 468 Thoth marks
Now for RMP 38. Ahmes comes up with a funny equation: 1 hekat x 3 '7 x '22 x 7 = 1 hekat
The hekat is a measure of capacity. 30 hekat equal 1 cubic cubit. 1 hekat may be defined as a right parallelepiped with the following measurements:
'2 royal cubit x '3 royal cubit x '5 royal cubit
or
28 Re marks x 28 Shu marks x 28 Geb marks
In Qhebsenuf marks:
210 Qm x 140 Qm x 84 Qm cubic diagonal exactly 266 Qm
according to the quadruple 6-10-15-19
A hekat in the shape of a right parallelepiped is well-defined. How about other shapes? Let us look again at Ahmes' equation:
1 hekat x 3 '7 x '7 x 22 = 1 hekat
The numbers 3 '7 and '7 of 22 remind us of pi and 1/pi. How about a hekat in the shape of a cylinder? I replace the left hekat with the above definition:
210 Qm x 140 Qm x 84 Qm x 3 '7 x '22 x 7 = 1 hekat
Now I transform my equation:
'4 x 105 Qm x 105 Qm x 3 '7 x '11 x 3136 Qm = 1 hekat
The first term
'4 x 105 Qm x 105 Qm x 3 '7 or '4 x 7 f x 7 f x 3 '7
can be regarded as the area of a circle whose diameter measures 105 Qhebsenuf marks or 7 fingers, while the second term
'11 x 3136 Qm = 19 '165 fingers
can be regarded as the height of a cylinder. I leave out the small fraction '165 and keep the height 19 fingers. Now I can define my cylindrical hekat and quadruple-hekat simply:

diameter	7 fingers	14 fingers
circumference	22 fingers	44 fingers
height	19 fingers	19 fingers
volume	1 hekat	4 hekat

The mistakes are tiny.

RMP 37 - a cone

Ahmes finds a volume that measures '4 '32 hekat or 90 ro. 1 cubic cubit equals 30 hekat. 1 hekat equals 320 ro. 90 ro can be given as a cone with these measurements (mistake '4 ro):
diameter base 7 fingers circumference 22 fingers height 4 palms

RMP 35 - a triangular pyramid

Another volume or capacity measures 96 ro = '100 of a cubic cubit. 96 ro can be given as a triangular pyramid whose base and height measure 13 and 9 fingers (mistake '131 ro).

```
 1       1       3
   2       4       6
   1       2       3
     3       5       9
       8      14      24
       4       7      12
         11      19      33
           30      52      90
           15      26      45
           15      26      45
             41      71      123
               112     194      336
                56      97      168
```

If the side of an equilateral triangle measures 194 parts, its height measures practically 168 parts. Check my above solution using these numbers. You will find the equation

195 x 195 equals 194 x 196 plus 1

RMP 21, 22 and 23 - Re had many names

By rolling a finely carved and polished stone disk on a carefully prepared ground one may find that the ratio circumference to diameter is less than 4, a little more than 3, less than 3 '6, and even a little less than 3 '7. These empirical values or boundaries can be used for generating many more approximate values for the number of the circle, which I call re, for the hieroglyph of the supreme sun god Re was a small circle, and while he had many names, no one knew his true one ...

You can easily see what I do:

```
 4  (plus 3)  7  10  13  16  19  22  25  28
 1  (plus 1)  2   3   4   5   6   7   8   9
```

If the diameter of a circle measures 7 palms or 1 royal cubit, the circumference measures practically 22 palms or 3 cubits 1 palm.

```
 6  (plus 19)  25  44  63
 2  (plus  6)   8  14  20
```

Solve RMP 37 using the value '20 of 63 = 3 '10 '20.

```
 9  (plus 19)  28  47  66  ...  256
 3  (plus  6)   9  15  21  ...   81
```

A famous formula of the Rhind Mathematical Papyrus says that a square with a side length of 8 royal cubits and a circle with a diameter of 9 royal cubits have roughly the same area. This formula is based on a value of 256/81 for pi.

```
 3  (plus 22)  25  47  69  91  113  135  157  179  201  223  245
 1  (plus  7)   8  15  22  29   36   43   50   57   64   71   78
267  289  311  333  355  377  399  421  443  465  487  509  531
 85   92   99  106  113  120  127  134  141  148  155  162  169
```

If the side of a square measures 10 royal cubits or 70 palms, the diagonal measures practically 99 palms, and the circumference of the circle around the square measures practically 311 palms. - If you wish to know the circumference of a circle, multiply the diameter by one of the following series:

```
 '99 of 311 = 3 '9 '33   '120 x 377 = 3 '8 '60 = 3 '10 '24
```

Yet another sequence:

```
 6  (plus 22)  28  50  72  ...  424  ...  600
 2  (plus  7)   9  16  23  ...  135  ...  191
 191/600 = '600 x 191 = '4 '30 '40 '100
```

If you know the circumference of a cylinder and wish to know its diameter, multiply the circumference by '4 '30 '40 '100.

```
 21  (plus 22)  43  65  ...  2463
  7  (plus  7)  14  21  ...   784 = 28x28
 286 (+311)…  132 (+333)…  333 (+355)…  201 (+377)… 2463
  91 (+ 99)…   42 (+106)…  106 (+113)…   84 (+120)…  784 = 28x28
```

If the radius of a circle measures 1 royal cubit or 28 fingers, the area of the circle measures practically 2463 square fingers.

Now for RMP 21-23:

```
 "3 '15            plus  '5 '15  equals  1
 "3 '30            plus  '5 '10  equals  1
 '4 '8 '10 '30 '45  plus  '9 '40  equals  "3
```

I multiply the first equation by a factor of 135, the second one by a factor of 50, and the third one by a factor of 360:

```
 99 + 36 = 135   35 + 15 = 50   161 + 49 = 240
```

Consider these numbers as diameters of nine circles. How long are their circumferences? Consulting the above number sequences you will derive the following solutions and values of re:

```
 c  47  22x5  113  22x7  157  311  424  600  377x2
 d  15   7x5   36   7x7   50   99  135  191  120x2
```

These values can easily be given as unit fraction series:

```
 '15 of 47   =  3 '10 '30      '7 of 22   =  3 '7
 '36 of 113  =  3 '9 '36      '50 of 157  =  3 '10 '25
 '99 of 311  =  3 '9 '33     '135 of 424  =  3 '15 '27 '135
'120 of 377  equals  3 '10 '24  or  3 '8 '60
'600 of 191  equals  '4 '30 '40 '100
```

RMP 36 - a pair of granaries

3 '3 '5 times '4 '53 '106 '212 equals 1

Imagine a square whose diagonal measures 2 royal cubits. Its area measures 2 square cubits.

Imagine a circle whose diagonal measures 3 royal cubits. Using the value '135 of 424 for re, the area of the circle measures 7 '15 square cubits.

Build a granary on the square (inner diagonal 2 royal cubits). Build a granary on the circle (inner diameter 3 royal cubits).

Fill the round granary to a height of 1 royal cubit. You will need 53 quadruple-hekat or 212 hekat of barley. Fill 212 hekat of grain in the square granary. The barley will reach a height of 3 '3 '5 royal cubits.

Fill the square granary to a height of 1 royal cubit. You will need 2 cubic cubits or 3 khar or 15 quadruple-hekat or 60 hekat of barley. Fill 60 hekat of barley in the round granary. It will reach a height of '4 '53 '106 '212 royal cubits.

RMP 31 - a granary on a ring

33 divided by 1 "3 '2 '7 equals 14 '4 '56 '194 '388 '679 '776

Imagine a regular hexagon whose side measures 66 fingers. Inscribe and circumscribe a circle. The two circles form a ring. The radius of the outer circle measures 66 fingers. How long is the radius of the inscribed circle in palms?

```
1        1        3
  2        4        6
  1        2        3
    3        5        9
    3        5        9
      8       14       24
      4        7       12
        11       19       33
          30       52       90
          15       26       45
            41       71      123
              112       194     336
               56        97    168
```

The side of the hexagon measures 66 fingers. Multiply 66 fingers by 168/97 and you obtain the diameter of the inscribed circle in fingers. Multiply 33 fingers by 168/97 and you obtain the radius in fingers. Divide 33 fingers by 97/42 = 1 "3 '2 '7 and you obtain the radius of the inscribed circle in palms:

33 fingers divided by 1 "3 '2 '7 equal 14 '4 '56 '97 '194 '388 '679 '776 palms

The area of the ring is given by the difference

area circumscribed circle minus area inscribed circle

The area of the circumscribed circle is found as follows:

radius x radius x re

66 fingers x 66 fingers x '99 x 311 = 13,684 square fingers

Now for the area of the inscribed circle. It measures

14 '4 '56 '194 '388 '679 '776 palms times 14 '4 '56 '194 '388 '679 '776 palms times re
Is anyone prepared to carry out that multiplication ???
Ahmes would smile and offer a much simpler solution based on a fine theorem:
Imagine a regular polygon of 3, 4, 5, 6, 7 ... equal sides. The circumscribed circle and the inscribed circle form a ring. Draw a circle around a side of the polygon. Its area equals the area of the ring.
The side of the regular hexagon measures 66 fingers, the radius of the circle around a side measures 33 fingers, and the area of the ring measures
33 fingers x 33 fingers x '99 x 311 = 3,421 square fingers
The area of the outer circle measures 13,684 square fingers, the area of the ring measures 3,421 square fingers, and the area of the inner circle measures 13,684 - 3,421 = 10,263 square fingers. Comparing these areas reveals the following proportions:
area inner circle / area ring / area outer circle = 3 / 1 / 4
Build a granary on the ring. If the height measures 5 '2 royal cubits or 154 fingers, the volume of the wall measures practically 24 cubic cubits, and the capacity 72 cubic cubits or 108 khar or 540 quadruple-hekat or 2160 hekat.

RMP 7 to 20 - spheres holding rectangles

RMP 7 '4 '28 times 1 '2 '4 equals '2

Imagine a rectangle measuring '4 '28 by 1 '2 '4 royal cubits or 8 by 49 fingers. Its area measures half a square cubit. Transform this area into a square, Its diagonal measures 1 royal cubit or 7 palms or 28 fingers. Draw a circle around the square. How long is its circumference?

```
1 royal cubit  x '7 x 22  equals  3 '7 royal cubits
7 palms        x '7 x 22  equal  22 palms
28 fingers     x '7 x 22  equal  88 fingers
```

The circumference measures 3 '7 royal cubits or 22 palms or 88 fingers (mistake less than one millimeter).
Partition the length of the rectangle 8 by 49 fingers according to the number 1 '2 '4. Thus you obtain three rectangles measuring

```
'4 '28 by 1  royal cubit  or  8 by 28 fingers
'4 '28 by '2 royal cubit  or  8 by 14 fingers
'4 '28 by '4 royal cubit  or  8 by  7 fingers
```

Imagine a sphere holding the rectangle 8 by 28 fingers and calculate the surface of the sphere using the formula: area circle = diameter x diameter x re. Choose a handy value of re from the number sequence

```
3  (plus 22)  25  47  69  91  113  ...  311  333  355  377
1  (plus  7)   8  15  22  29   36  ...   99  106  113  120
```

The diameter squared is found as follows:

```
8x8 plus 28x28  =  64 plus 784  =  848  =  8 x 106
```

Using the value '106 of 333 for re we obtain

```
8 x 106 x '106 x 333  =  2,664 square fingers
```

The surface of the sphere around the rectangle 8 by 28 fingers measures 2,664 square fingers (mistake 25 square millimeters). Imagine a sphere holding the rectangle 8 by 7 fingers and calculate its surface in the same way:

```
8x8 plus 7x7 equals 113   113 x '113 x 355 equals 355
```

The surface of the sphere around the rectangle 8 by 7 fingers measures 355 square fingers (mistake '94 square millimeter).

Imagine a sphere holding the long rectangle 8 by 49 fingers and calculate the surface again, this time using the values '99 of 311 = 3 '9 '33 and '120 of 377 = 3 '8 '60. Round all the numbers:

```
 8x8 plus 49x49  equals  2465
2465 x 3 '9 '33  --  7395  274  75  =  7744 square fingers
2465 x 3 '8 '60  --  7395  308  41  =  7744 square fingers
```

The surface of the sphere holding the rectangle 8 by 49 fingers measures practically 7744 square fingers (mistake 9 square mm). Thus we found a new value for re: '2465 of 7744

```
289  (plus 355)  649  999 ... 2419 ... 5969 ... 7389  7744
 92  (plus 113)  205  318 ...  770 ... 1900 ... 2352  2465
```

Transform the area 7744 square fingers into a square:

```
7744 square fingers = 88 fingers by 88 fingers
```

The side of the square measures 88 fingers (mistake '362 mm). The sphere around the rectangles 8 by 49 fingers holds another rectangle that measures 23 by 44 fingers. Diameter squared:

```
8x8 plus 49x49  =  23x23 plus 44x44  =  2465 square fingers
```

The square 44 by 44 fingers and a circle around the rectangle 23 by 44 fingers have practically the same area:

```
44x44 = 23x23 plus 44x44 x '2465 x 7744 = 1936 square fingers
```

If you wish to transform a square into a circle of the same area you may shorten the square from 44 to 23 equal parts and draw a circle around the resulting rectangle. - A fairly good approximation is obtained by simply bisecting the square. This solution is based on the value '81 of 256 for re.

RMP 8 '4 times 1 "3 '3 equals '2

A rectangle measures 7 by 56 fingers. The imaginary circle around this rectangle circumscribes another rectangle that measures 28 by 49 fingers or 1 by 1 '2 '4 royal cubits.

.

```
7x7 plus 56x56 = 28x28 plus 49x49  (diameter squared)
```

The periphery of the rectangle 28 by 49 fingers measures 154 fingers. Transform the periphery into the circumference of a circle. Use the value '7 x 22 for re. You will obtain 49 fingers or 1 '2 '4 royal cubits.

```
 154 f x '22 x 7  =  49 fingers or 1 '2 '4 royal cubits
```

RMP 9 '2 '14 times 1 '2 '4 equals 1

A rectangle measures 16 by 49 fingers. Imagine a sphere holding this rectangle. The sphere also holds a right parallelepiped that measures 28 by 28 by 33 fingers. The face 28 by 28 fingers represents a square cubit and corresponds to the area of the long rectangle measuring 16 by 49 fingers.

```
 16x16 + 49x49 = 28x28 + 28x28 + 35x35  (diameter squared)
```

RMP 10 '4 '28 times 1 '2 '4 equals '2 (see RMP 7)

A rectangle measures 8 by 49 fingers. The sphere around this rectangle holds a right parallelepiped that measures 1 royal cubit x 10 palms x 9 fingers = 28 by 40 by 9 fingers. The narrow faces measure 9 by 40 fingers each; their diagonals measure exactly 41 fingers, according to the triple 9-40-41.

```
 8x8 + 49x49 = 9x9 + 28x28 + 40x40    9x9 + 40x40 = 41x41
```

Imagine a sphere holding the rectangle 9 by 40 fingers and calculate its surface using the series 3 '9 '33 and 3 '8 '60:

```
  9x9 plus 40x40  =  41x41  =  1681
 1681 x 3 '9 '33  --  5043  187  51  =  5281
 1681 x 3 '8 '60  --  5043  210  28  =  5281
```

A new value for the number re: '1681 of 5281

```
 311  (plus 355)  666  1021  ...  4926 (2463)  5281  5636
  99  (plus 113)  212   325  ...  1568  (784)  1681  1794
```

RMP 11 '7 times 1 '2 '4 equals '4

A rectangle measures 4 by 49 fingers. The imaginary sphere around this rectangle holds a right parallelepiped that measures 14 by 14 by 45 fingers. Volume or capacity 12 '28 '56 hekat. Using the value '20 of 63 for re, this volume can be transformed into a cylinder whose diameter, circumference and height measure 5 palms, 63 fingers and 1 royal cubit.

RMP 12 '14 times 1 '2 '4 equals '8

A rectangle measures 2 by 49 fingers. The sphere around it holds a right parallelepiped that measures 20 by 22 by 39 fingers. The numbers 22 and 39 provide a good value for the square root of 3 '7 or '7 of 22.

```
 39 x 39 x 7  plus 1  =  22 x 22 x 22
```

RMP 13 '16 '112 times 1 '2 '4 equals '8

A rectangle composed of six rectangles measures 2 by 49 fingers (see RMP 12). The smallest rectangle measures '112 by '4 royal cubit and has an area of 1 '2 '4 square fingers. The largest rectangle measures '16 by 1 royal cubit and has an area of 49 square fingers (RMP 14).

RMP 14 '28 times 1 '2 '4 equals '16

A rectangle measures 1 by 49 fingers. A square that measures 7 by 7 fingers has the same area. Imagine a sphere around the square and calculate its surface. Using the value 3 '7 for re you will obtain 308 square fingers.

```
 7x7 plus 7x7  times  3 '7  equals  308
```

RMP 15 '32 '224 times 1 "3 '3 equals '14

A rectangle measures 1 by 56 fingers. The rectangles 4 by 14 fingers and 8 by 7 fingers have the same area. Draw a circle around the rectangle 4 by 14 fingers and imagine a hemisphere standing on the circle. Its surface measures practically 333 square fingers. Now imagine a sphere holding the rectangle 8 by 7 fingers. The surface measures practically 355 square fingers.

RMP 16 '2 times 1 "3 '3 equals 1

A rectangle measures 14 by 56 fingers and has the same area as a square that measures 7 by 7 palms or 28 by 28 fingers. Imagine a sphere around the square and calculate its surface. Using the value 3 '7 for re, the surface measures 308 square palms. Using the value '784 of 2463 for re, the surface measures 2463 square fingers.

RMP 17 '3 times 1 "3 '3 equals "3

A triple square, a double square and a square measure '6 by 1 square cubit, '6 by "3 square cubit, and '6 by '3 square cubit. The diagonals of the square, of the double square and of the triple square can be approximated by means of my number columns that begin with the lines 1-1-2 and 1-1-5. Draw a circle around each figure. Their areas will maintain the ratio 2:5:10.

RMP 18 '6 times 1 "3 '3 equals '3

The area of a rectangle measures '3 square cubit. Transform this area into a square. Imagine a sphere holding the square. Calculate its surface using the value '784 of 2463 for re. The diameter squared measures '3 plus '3 = "3 square cubits. Multiply this area by 2463 and you obtain 1642 square fingers.

RMP 19 '12 times 1 "3 '3 equals '6

The area of a rectangle measures '6 square cubit. Transform this area into a square. Imagine a sphere holding the square. Calculate the surface of the sphere:

```
'6 '6 square cubits  x  2463  =  821 square fingers
```

A highly demanding task, solvable in a very simple manner (mistake only one square millimeter).

RMP 20 '24 times 1 "3 '3 equals '12

The area of a rectangle measures '12 square cubit. Transform it into a square. Imagine a sphere holding the square. Transform the surface of the sphere into a circle. Imagine a second sphere holding the circle. It will be the sphere from RMP 18, whose surface measures practically 1642 square fingers.

RMP 24 to 30 - Sacred Triangle 3-4-5 (cube 1-1-1)

These problems are written in a column taking up the whole height of the sheet it is written on, and may concern the Sacred Triangle 3-4-5, the inscribed circle, the cube 1-1-1, and half a cube.
Imagine a circle inscribed in a rectangular triangle. How long is the diameter? Use this very simple formula: diameter = sum of legs minus hypotenuse. Consider the case of the Sacred Triangle measuring 3-4-5 palms. The diameter of the inscribed circle measures

3 plus 4 minus 5 = 2 palms

RMP 24 A plus '7 A equals 19

Imagine a Sacred Triangle measuring 9 units (base), 12 units (height), 15 units (slope), 3 units (radius of the inscribed circle), and 6 units (diameter of the inscribed circle):

base + height + radius = 19 palms

Base, height and slope measure 7 '8 - 9 '2 - 11 '2 '4 '8 palms. The diameter of the inscribed circle measures 19 fingers.

RMP 25 A plus '2 A equals 16

height + diameter = 16 palms

Base, height, slope and diameter measure 8 - 10 ''3 - 13 '3 5 '3 palms. The periphery measures 24 palms.

RMP 26 A plus '4 A equals 15

height + radius = 15 palms

Base, height, slope and diameter measures 9 - 12 - 15 - 6 palms.

RMP 27 A plus '5 A equals 21

slope + diameter = 21 palms

Base, height, slope and diameter measure 42 - 56 - 70 - 28 fingers. The diameter of the inscribed circle measures 1 royal cubit. The height of the triangle measures 2 royal cubits, and the periphery measures 6 royal cubits.

Calculate the circumference of the inscribed circle using the formula circumference = diameter x re and compare it to the periphery of the Sacred Triangle. The ratio equals re to 6 (about 1/2 or 23/44 or 67/128 or 111/212). Calculate the area of the inscribed circle using the formula '4 x diameter x diameter x re and compare it to the area of the Sacred Triangle. You will find the same ratio: re to 6.

Imagine a sphere in a cube, calculate the volume of the sphere using the formula '6 x diameter x diameter x diameter x re and compare it to the volume of the cube. The ratio equals re to 6 again. Calculate the surface of the sphere using the formula diameter x diameter x re and compare it to the surface of the cube. The ratio equals re to 6.

Compare the Sacred Triangle 3-4-5 palms to the cube 1-1-1 palms. Both the periphery of the triangle and the sum of the edges of the cube measure 12 palms. Both the area of the triangle and the surface of the cube measure 6 square palms. And both the area of the circle in the triangle and the surface of the sphere in the cube measures re square palms.

RMP 29 10 A times 1 '4 '10 equals 13 '2

The diagonal of a square measures 10 palms. The side of the square equals the circumference of a circle inscribed in a Sacred Triangle. Calculate the periphery of this triangle.

```
1        1        2
  2        3        4
    5        7       10
      12       17       24
        29       41       58
          70       99      140
```

Multiply 10 palms by '99 x 140, then by '22 x 7, then by 6:

```
10 x '99 x 140 x '22 x 7 x 6 = 10 x 1 '4 '10 = 13 '2
```

The periphery of the Sacred Triangle measures 13 '2 palms. Base, height, slope and diameter measure 3 '4 '8 - 4 '2 - 5 '2 '8 - 2 '4 palms.

RMP 30 A times "3 '10 equals 10

Imagine a Sacred Triangle whose periphery measures 10 royal cubits or 70 palms Calculate the distance of the center of the inscribed circle from the corner between base and slope in palms:
The periphery of the triangle measures 10 cubits or 70 palms. Base, height and slope measure 17 '2 - 23 '3 - 29 '6 palms. Radius and diameter of the inscribed circle measure 5 '2 '3 and 11 "3 palms. The distance in question is the slope of a rectangular triangle whose rise and run measure 5 '2 '3 and 11 "3 palms, corresponding to the diagonal of a double square measuring 5 '2 '3 by 11 "3 palms.

```
 1        1        5
   2        6       10
   1        3        5
     4        8       20
     2        4       10
     1        2        5
       3        7       15
         10       22       50
          5       11       25
           16       36       80
            8       18       40
            4        9       20
              13       29       65
                42       94      210
                21       47      105
                  68      152      340
                  34       76      170
                  17       38       85
                    55      123      275
                     178      398      890
                      89      199      445
                       288      644     1440
                       144      322      720
                        72      161      360
```

Multiply 5 '2 '3 palms by 360 and you obtain 2100 palms. Divide 2100 palms by 161 and you obtain 13 '23 palms. Solution of RMP 30: 13 '23 (palms). Check the result. Multiply 13 '23 palms by '360 x 161, thus you obtain the radius of the circle in palms. Multiply it by 12 and you obtain the periphery of the triangle in palms. Multiply it by '7 and you obtain the periphery in royal cubits:

```
 13 '23 x '360 x 161 x 12 x '7 = 13 '23 x "3 '10 = 10
```

Half a cube measures 5 '2 '3 by 11 "3 by 11 "3 palms. The diagonals of the faces measure practically 13 '23 palms and 66 fingers respectively. The cubic diagonal measures exactly 70 fingers, according to the quadruple 1-2-2-3. The four cubic diagonals together measures 10 royal cubits.
If you were told to imagine half a cube, you were informed that the sum of the cubic diagonals measures 10 royal cubits, and you were asked to calculate the diagonal of a smaller face in palms, you would have to carry out the following multiplication

```
 10 times '4 x 7 x '3 x 360 x '161 equals A
```

or to solve this equation

```
 A times 4 x '7 x 3 x '360 x 161 equals 10
```

corresponding to Ahmes' equation A times ''3 '10 equals 10 (RMP 30).

RHIND MATHEMATICAL PAPYRUS, problems no. 39, 40, 64 (intermezzo)

RMP 39

100 loaves of bread are distributed among ten men. 50 loaves are given to 5 men. Each man obtains 12 '2 loaves. The remaining 50 loaves are given to 6 men. Each man receives 8 '3 loaves. Now Ahmes calculates the difference

```
 12 '2  minus  8 '3  equals 4 '6
```

Let me continue as follows:

```
 '2 x 25  minus  '3 x 25  equals  '6 x 25
 '2 minus '3 equals '6
  '2 equals '3 plus '6
```

This equation belongs to a stairway:

```
'2 = '2
'2 = '1x3 '6
'2 = '1x3 '3x5 '10
'2 = '1x3 '3x5 '5x7 '14
'2 = '1x3 '3x5 '5x7 '7x9 '18
'2 = '1x3 '3x5 '5x7 '7x9 '9x11 '22
....................................
'2 = '1x3 '3x5 '5x7 '7x9 '9x11 '11x13 '13x15 '15x17 '17x19 ...
```

RMP 40

100 loaves of bread are distributed as follows: 3 men obtain 23 ''3 '7 loaves each, and 2 men receive 14 '4 '28 loaves each. Let me proceed in the same way as before:

```
 23 ''3 '7  minus  14 '4 '28  equals  9 '2 '42
 '42 x 1000  minus  '70 x 1000  equals  '105 x 1000
 '42 minus '70 equals '105
 '6 minus '10 equals '15
 '6 equals '10 plus '15
```

This equation belongs to another number pattern:

```
 '2  =  '6   '1x3
 '6  =  '10  '3x5
'10  =  '14  '5x7
'14  =  '18  '7x9
'18  =  '22  '9x11   (and so on)
```

RMP 64

Ten men obtain 7, 9, 11, 13, 15, 17, 19, 21, 23, 25 parts of barley, all in all ten hekat. The volume of one part can be defined as follows:

```
 7 fingers  x  7 fingers  x  14 Qhebsenuf marks
   (15 Qhebsenuf marks equal 1 finger)
```

15 parts can be defined as follows: 7 by 7 by 14 fingers

RHIND MATHEMATICAL PAPYRUS, problems no. 41- 60 (demanding)

RMP 41

A granary in the form of a cylinder has an inner diameter of 9 royal cubits and an inner height of 10 royal cubits.

A square of the side 8 rc and a circle of the diameter 9 rc have about the same area. So the floor of the granary measures about 64 square cubits. Multiply this area by the height 10 royal cubits and you will obtain a volume of 640 cubic cubits = 960 khar = 4,800 quadruple hekat = 19,200 hekat.

Now let us calculate the volume of the granary more exactly:

```
 diameter   9 royal cubits  or  63 palms  or  252 fingers
 height    10 royal cubits  or  70 palms  or  280 fingers
```

By using the value '7 of 22 for re we obtain:

```
 diameter         63 palms  or   252 fingers
 circumference   198 palms  or  1386 fingers
 area wall 13,860 square palms  area floor 49,896 square fingers
 volume  13,970,880 cubic fingers or about 636 cubic cubits
```

A better result than the first one. Now, as a game for advanced learners, we may exchange the numbers:

```
 diameter  10 royal cubits  or  70 palms  or  280 fingers
 height     9 royal cubits  or  63 palms  or  252 fingers
 circumference                 220 palms  or  880 fingers
 area wall  13,860 square palms
 area floor  3,850 square palms  or  61,600 square fingers
 volume  15,523,200 cubic fingers  or  about 707 cubic cubits
```

The walls of the two cylinders have the same area while their volumes maintain the ratio 9 to 10.

The diameter of another cylinder measures 15 royal cubits and the height 6 royal cubits. The wall again has the same area while the volume increases to 23,284,800 cubic fingers or about 1061 cubic cubits.

A further cylinder may have a diameter of 90 royal cubits and a height of 1 royal cubit. The area of the wall would again be the same while the volume would increase to about 6364 cubic cubits.

Now please consider a sequence of cylinders:

diameter	1	2	3	5	6	9	10	15	18	30	45	90	rc
height	90	45	30	18	15	10	9	6	5	3	2	1	rc

The walls have the same area while the volume increases in a peculiar way: divide the height by any number and the volume will increase by the same factor ... The lower the cylinder the bigger the volume. And if the wall has no height at all? In that case the volume would be infinitely huge ... A pretty paradox to be discussed in the seminary of professor Ahmes.

RMP 42

The inner diameter of another cylindrical granary measures 10 royal cubits and the inner height again 10 royal cubits. Using the same formula (diameter 9, side 8), Ahmes obtains a capacity of 790 '18 '27 '54 or 790 '9 cube cubits instead of 790 '9 '81 cube cubits. Using the value '7 of 22 for re we would obtain 785 '2 '7 '14 cubic cubits. Using the still better value '120 of 377 we obtain 785 '3 '12 cube cubits.

diameter	1	2	4	5	10	20	25	50	100
height	100	50	25	20	10	5	4	2	1

The walls of these cylinders have the same area, while the volume increases from about 78 '2 to about 7854 cube cubits. Let us consider the following pair of examples:

```
diameter  2   height 50   volume  157   re   '50 of 157
diameter 50   height  2   volume 3927   re '1250 of 3927
```

The second value for re is generated by a number sequence:

```
33  (plus 66)  99  165  ...  3795  3861  3927
11  (plus 21)  32   53  ...  1208  1229  1250
```

The same value can be found by means of two multiplication processes:

```
1250 x 3 '9 '33   equals  3750  139  38  or  3927 (rounded)
1250 x 3 '10 '24  equals  3750  125  52  or  3927 (rounded)
```

RMP 43

Ahmes calculates the capacity of a cylindrical granary whose inner diameter measures 6 royal cubits while the inner height measures 9 royal cubits. However, his result is wrong, because in the middle of his calculation he jumps from one formula to a different one. Perhaps an intentional mistake to challenge his pupils? Here I am not concerned with any mistake or possible intentions, but only with the numbers of the cylinder.

If a circle of the diameter 9 rc and a square of the side 8 rc have the same area, the volume of the above cylinder measures 256 cubic cubits. Now let me compare two cylinders:

```
diameter   6 or 9 royal cubits
height     9 or 6 royal cubits
```

If you remember my interpretation of RMP 41, you can easily calculate the measurements of the second cylinder: its round wall has the same area while its volume equals

```
 '6 x 9 x 256 cubic cubits  =  384 cubic cubits
```

Now please imagine a granary in the shape of a hemi-ellipsoid within the frame of the second cylinder: what is the volume?

The circle of the second cylinder has a diameter of 9 royal cubits. Let us first imagine a sphere with the same diameter, 9 rc, and calculate its volume by means of the formula

volume sphere = '6 diameter x diameter x diameter x re

By using the value '81 of 256 or '81 x 256 for re we obtain:

```
 '6 x 9 x 9 x 9 x '81 x 256 ccc  =  384 cubic cubits
```

What a tidy result: a cylinder with diameter 9 and height 6 and a sphere with diameter 9 have exactly the same volume.

Now for the ellipsoid. This geometric form is nothing other than a sphere lengthened in one dimension. The diameter of the above sphere measures 9 cubits in every dimension while the vertical diameter of the ellipsoid measures 6 + 6 = 12 cubits. We obtain the volume of the ellipsoid by multiplying that of the sphere by a factor of '9 x 12 = '3 x 4 as follows:

```
 sphere      diameter 9 x 9 x 9     volume  384 ccc
 ellipsoid   diameter 9 x 9 x 12    volume  512 ccc
```

Now the volume of the hemi-ellipsoid equals '2 x 512 ccc = 256 cubic cubits: exactly the volume of the first cylinder.

A granary in the shape of a cylinder has a diameter of 6 royal cubits and a height of 9 royal cubits -- another granary in the shape of a hemi-ellipsoid has a diameter of 9 royal cubits and a height of 6 royal cubits -- the two granaries have exactly the same volume

RMP 44 and 45

These problems concern a container in the form of a cube measuring 10 by 10 by 10 royal cubits. If you wish to build such a granary you have to know the lengths of the floor diagonals, of the face diagonals, and of the cubic diagonals.

```
 1       1       2
   2       3       4
     5       7      10
       12      17      24
         29      41      58
           70      99     140   and so on
```

If the edge of a cube measures 10 royal cubits or 70 palms, the diagonal of a face measures practically 99 palms (mistake 0.4 millimeters).

```
 1       1       3
   2       4       6
   1       2       3
```

```
  3       5       9
    8       14      24
    4        7      12
      11      19      33
        30      52      90
        15      26      45
          41      71     123
            112     194     336
             56      97     168   and so on
```

If the edge of a cube measures 10 royal cubits or 70 palms or 280 fingers = 5 x 56 fingers, the cubic diagonal measures practically 5 x 97 = 485 fingers (mistake 0.5 millimeters).
Now we may go a step further and imagine a cone, a sphere, a hemi-ellipsoid and a cylinder in the frame of the cube. By using the simple value '50 of 157 for re we obtain:

```
CONE              diameter base   10 royal cubits
       area base       78 '2 square cubits
       height          10 royal cubits
       volume         261 "3 cubic cubits
SPHERE            diameter        10 royal cubits
       volume         523 '3 cubic cubits
HEMI-ELLIPSOID    diameter base   10 royal cubits
       area base       78 '2 square cubits
       height          10 royal cubits
       volume         523 '3 cubic cubits
CYLINDER          diameter        10 royal cubits
       area base       78 '2 square cubits
       height          10 royal cubits
       volume         785 cubic cubits
Volumes  CONE : HEMI-ELLIPSOID : CYLINDER  =  1 : 2 : 3
```

The granary in the form of a cube has a volume of 1,000 cubic cubits, while a granary in the shape of a hemi-ellipsoid in the frame of the same cube has a volume of about 523 cubic cubits. When we transform the cube into a right parallelepiped, the ratio of the volume of the parallelepiped to the volume of the inscribed hemi-ellipsoid remains the same, about 2. This generates a very simple practical formula:
If you see a granary in the shape of a hemi-ellipsoid and wish to estimate its volume, carry out the following calculation: '2 x diameter base x diameter base x height and you obtain the approximate volume of the granary.

RMP 46 and 47

Two square containers measure

```
 10 by 10  by  3 '3  royal cubits
 10 by 10  by  5     royal cubits
```

I am assuming that the floor of each container measures 10 by 10 royal cubits. Now let me inscribe an octagon in that square:

```
    A    a      b   B
    h               c
    g               d
    D    f      e   C
Square ABCD   AB = BC = CD = DA = 10 royal cubits
inscribed octagon abcdefgh   ab=bc=cd=de=ef=fg=gh=ha
```

A side of the square measures 10 royal cubits or 70 palms or 280 fingers. How long is the side of the regular octagon within the frame of the square?

```
1       1       2
  2       3       4
    5       7      10
      12      17      24
        29      41      58
          70      99     140
            169      ...      ...
side of octagon     side of circumscribed square
   7             5  +   7  +   5   =   12
   10             7  +  10  +   7   =   24
   17            12  +  17  +  12   =   41
   24            17  +  12  +  17   =   58
   41            29  +  41  +  29   =   99
   58            41  +  58  +  41   =  140
```

By doubling the numbers of the last line we find:

```
side of octagon  116    side of square  82+116+82 = 280
```

The side of the square measures 10 royal cubits or 70 palms or 280 fingers, hence the side of the inscribed octagon measures practically 116 fingers or 4 cubits 1 palm.

Now let me go a step further and transform the square containers into granaries of the same volume but in the shape of prisms. The floor of these containers is given by the above octagon. The area of an octagon is smaller than that of the square. Accordingly, the granaries will be taller. How do we calculate their heights? Again by means of the above number pattern:

```
height of granary based on square    10  24  58  140  ...
height of granary based on octagon   12  29  70  169  ...
```

The former square containers were 3 '3 and 5 royal cubits tall. The new granaries on the base of the octagon are taller. By using the first numbers 10 and 12 we find the heights:

```
3 '3 royal cubits x '10 x 12  =  4 royal cubits
5 royal cubits    x '10 x 12  =  6 royal cubits
```

Very simple numbers. - Now let us consider only the second granary. Wishing to get a more accurate volume I use the numbers 140 and 169:

```
former height  5 royal cubits or 140 fingers
```

```
new height  140f x '140 x 169 = 169f = 6 cubits 1 finger
```

Calculating the area of the inner wall (right parallelepiped and prism):

```
base  10 c x 10 c     height  5 royal cubits
periphery  4 x 10 c  =  40 royal cubits
area of wall  40 c x 5 c  =  200 square cubits
side of octagon  116 fingers or 4 cubits 1 palm
height  169 fingers or 6 cubits 1 finger
periphery  8 x 116 fingers = 928 fingers or 33 cubits 1 palm
area of wall  =  33 '7 c x 6 '28 c  =  200 '28 '196 cc
```

The walls of the two granaries have practically the same area. Even better: they have exactly the same area (the minor error in the above result is due to the margins of error in the values '41 of 58 and '140 of 169 used for the calculation of the octagon).

Imagine a pair of ideal granaries with vertical walls. One granary is based on a square. The other granary is based on the regular octagon inscribed in the square. If the granaries have the same capacity, their walls have the same area. Or if the inner walls have the same area, the two granaries have the same volume.

RMP 48

Problem no. 48 of the Rhind Mathematical Papyrus contains a famous drawing of a square with an inscribed octagon:

```
    A a b B
    h     c
    g     d
    D f e C
Square ABCD  irregular octagon abcdefgh
A-B = B-C = C-D = D-A = 9 royal cubits
A-a = a-b = b-B = B-c = ... = g-h = h-A = 3 royal cubits
grid  3+3+3 by 3+3+3 royal cubits
area square  9x9 = 81 square cubits
area octagon  9x9 - 2x3x3 = 63 square cubits
```

A circle inscribed in the square would have about the same area as the octagon. This generates the value 3 '9 for re:

```
'4 x 9 rc x 9 rc x 3 '9  =  63 square cubits
```

63 square cubits are about 8 by 8 royal cubits. From this we derive a well known formula: if the diameter of a circle measures 9 units and if the side of a square measures 8 units the circle and the square have roughly the same area. This formula generates the value '81 of 256 for re, or nearly '6 of 19 or 3 '6, according to a crosswise multiplication:

```
256 x 6 = 1536    81 x 19 = 1539
```

So we found the values 3 '9 and 3 '6. The equally simple values in between are 3 '8 and 3 '7

RMP 49

A rectangle measures 2 by 10 khet or 200 by 1,000 royal cubits while its area measures 200,000 square cubits. Can you transform the rectangle into a regular octagon of about the same area?

```
circumscribed square  492 by 492 royal cubits
partition  12 times 12+17+12 royal cubits
grid  144+204+144 by 144+204+144 royal cubits
side of octagon  204 royal cubits
area  200,592 square cubits
```

RMP 50

Ahmes calculates the area of a circle whose diameter measures 9 khet = 900 royal cubits. By using his well known formula he obtains 8 by 8 khet = 64 square khet = 64 aroures or setat = 640,000 square cubits.

Advanced learners may try to solve a more demanding task: transforming the circle into a regular octagon of the same area using the following extended number pattern:

```
1       1       2       2
  2       3       4       6
    5       7      10      14
      12      17      24      34
        29      41      58      82
          70      99     140     198
            169     239     338     478
```

The squared side of a regular octagon and the area of the same octagon maintain a relation that can be approximated by means of the above numbers:

```
side x side   12  17   29   41   70   99  ...  square cubits
area octagon  58  82  140  198  338  478  ...  square cubits
```

The number re may be chosen from the following sequence:

```
3  (plus 22)  25  47  69  91  113  135  157  179  201  223
1  (plus  7)   8  15  22  29   36   43   50   57   64   71
245  267  289  311  333  355  377  399
 78   85   92   99  106  113  120  127
```

Two values contain the number 99: '99 of 478 and '99 of 311. Now the area of a regular octagon and the one of a circle may be defined like this:

```
side x side x '99 x 478    radius x radius x '99 x 311
```

The octagon and the circle have the same area, therefore:

```
side x side x '99 x 478  =  radius x radius x '99 x 311
```

The diameter of the circle measures 9 khet or 900 royal cubits while the radius measures 450 royal cubits. Now we obtain:

```
side x side  =  450 cubits x 450 cubits x 311 x '478
side x side  =  practically 131,752 square cubits
```

By consulting a table of square numbers you will find

```
362 x 362  =  131,044  ---       708 less than 131,752
363 x 363  =  131,769  ---  only  17 more than 131,752
364 x 364  =  132,496  ---       744 more than 131,752
```

The number 363 is a good solution to our problem. Hence a circle of the diameter 9 khet and a regular octagon of the side length 363 royal cubits have practically the same area.

```
grid  770+1089+770 by 770+1089+770  '3 royal cubits
```

RMP 51

Ahmes calculates the area of a triangle whose base measures 4 khet and its height 10 khet, generating an area of 20 square khet = 20 aroures = 200,000 square cubits. This area can be transformed into a regular octagon (see RMP 49):

```
grid  144+204+144 by 144+204+144 royal cubits
side  204 royal cubits   area  200,592 square cubits
```

The drawing of RMP 51 shows a triangle a base measuring 4 khet, while its height measurement is given as 13 khet. In this case the area measures 260,000 square cubits, while the regular octagon of roughly the same area has the following measurements:

```
grid  164+232+164 by 164+232+164 royal cubits
side  232 royal cubits   area  259,808 square cubits
```

RMP 52

This problem concerns a trapezoid whose base measures 6 khet, its upper side 4 khet and its height 20 khet. Its area is 100 aroures or 1,000,000 square cubits. Transform that area into a regular octagon by using an alternative number pattern:

```
2       1       4
  3       5       6
    8      11      16
      19      27      38
        46      65      92
       322     455     644
grid  322+455+322 by 322+455+322 royal cubits
side octagon  455 royal cubits   area  1,000,433 sc
```

RMP 53 (highly demanding)

The drawing of RMP 53 shows an isosceles triangle with a pair of additional lines parallel to the base. The numbers are (P = peak, AL = additional line, B = base):

```
 7 (P)  7 '2 '4 '8   2 '4 (AL)  3 '4   6 (AL)  5   6 (B)
```

The additional lines mark a trapezoid with these measurements:

```
top       2 '4 khet (225 royal cubits)
height    3 '4 khet (325 royal cubits)
base      6    khet (600 royal cubits)
```

Advanced learners may answer the following questions: a) how long are the oblique sides? b) how long are the sides of the regular octagon of the same area? Let us multiply the above numbers by a factor of 8:

```
top       18          18
height    26
base      48       15+18+15
```

The trapezoid is composed of a rectangle of 26 x 18 units and a pair of two rectangular triangles measuring 15 units (base) and 26 units (height). By removing the rectangle and joining the pair of triangles we obtain a triangle. Its base measures 30 units while the height measures 26 units. How long are its slopes? They measure practically 26 units again, according to the following number pattern, which can be used both for approximating the diagonal of a cube and the side and height measurements of an equilateral triangle:

```
1       1       3
  2       4       6
  1       2       3
    3       5       9
      8      14      24
      8      14      24
      4       7      12
        11      19      33
          30      52      90
          15      26      45
            41      71     123
             112     194     336
              56      97     168  and so on
```

These numbers approximate the equilateral triangle:

```
half side   4    7   15   26    56    97   ...
height      7   12   26   45    97   168   ...
side        8   14   30   52   112   194   ...
```

Now the trapezoid of RMP 53 can be defined as follows:

```
top         18
slope       30
height      26
base        48    (angles 60 and 120 degrees)
```

We answered the first question. Now for the second task. I multiply the above numbers by a factor of 3 and obtain:

```
top 54   height 78   slope 90   base 144
```

The area of the trapezoid is found as follows:

```
(base + upper side) x height x '2
 (144 + 54)         x   78   x '2  =  198 x 39
```

The regular octagon of about the same area can be approximated by means of these numbers:

```
1       1       2       2
  2       3       4       6
    5       7      10      14
      12      17      24      34
        29      41      58      82
          70      99     140      198  and so on
side x side      12  17   29   41  ...
area octagon     58  82  140  198  ...
```

The area of the trapezoid is given by the product 39 x 198, while the area of the octagon may be defined thus:

```
side x side x 198 x '41
```

The octagon and the trapezoid have the same area, therefore:

```
side x side x 198 x '41  =  198 x 39
side x side x '41 = 39   side x side = 39 x 41 = 1599
side of octagon  =  practically 40
```

Now let me divide all numbers by 3 x 8 = 24 in order to obtain measurements in khet:

```
top trapezoid        2 '4
height trapezoid     3 '4
slope trapezoid      3 '2 '4
base trapezoid       6
side octagon         1 "3
```

The other numbers of the drawing may represent isosceles triangles of the following measurements:

```
height   5    7 '2 '4 '8    7              7
base     6    2 '4          7 '2 '4 '8     6
```

I transform the triangles into octagons of about the same area:

base	height	factor	octagon side
6	5	21	37 or 1 '2 '4 '84
6	7	12	25 or 2 '12
2 '4	7 '2 '4 '8	12	23 or 1 '2 '3 '12
7 '2 '4 '8	7	72	172 or 2 '3 '18

In problem no. 53 of the Rhind Mathematical Papyrus we may assemble 8 or more similar problems, which can hopefully be reconstructed in the context of the previous calculations. The drawing of RMP 53 led me to a trapezoid and four triangles whose areas are quite easily transformed into regular octagons. However, I overlooked one possibility. The numbers

```
  7     7 '2 '4 '8     2 '4
```

define a triangle with a base of 2 '4 khet and a height of 7 khet with an area measuring 7 '2 '4 '8 square khet or setat (or aroure, Greek name for setat). This area is seen in the last part of the written calculation:

```
 2 '4 x 7 setat x '2  =  7 '2 '4 '8 setat
```

Now let me transform this area, 7 '2 '4 '8 setat or 78,500 square cubits, into a regular octagon:

```
 side x side      12  17   24   29  ...
 area octagon     58  82  116  140  ...
 78,750 x 29 x '140  =  16,312 '2

 127 x 127 = 16,129     128 x 128 = 16,384
```

No simple numbers. Let me begin anew and try it with palms instead of royal cubits:

```
 700 x 7 x 225 x 7 x '2  =  3,858,750 (square palms)
  3,858,750 x 29 x '140  =  799,312 '2
       893 x 893  =  797,449  (1863 '2 less)
       894 x 894  =  799,236  (  76 '2 less)
       895 x 895  =  801,025  (1712 '2 more)
```

A triangle with a base of 225 royal cubits and a height of 700 royal cubits and a regular octagon whose side measures 894 palms have roughly the same area. Now let me look for a suitable grid. The number 894 does not appear in the above number pattern, but we may proceed like this:

```
 6 x 140  plus  41  plus  10  plus  3  =  894
 6 x  99  plus  29  plus   7  plus  2  =  632
 9 x 99  plus  3  =  894
 9 x 70  plus  2  =  632
```

Hence the grid of the octagon of the same area measures roughly

```
 632+894+632 by 632+894+632 palms
```

while the octagon's area measures 3,858,116 square palms or practically 78,737 square cubits, only 13 square cubits less than 78,750 square cubits.

One of RMP 53's calculations seems to concern a triangle with the following measurements:

```
base  2 '4 khet  (225 rc)   height  4 '2 khet (450 rc)
area    '2 x 225 cubits x 450 cubits  =  50,625 setat
(1 setat being 1 square khet or 10,000 square cubits)
```

Transforming this area into a regular octagon:

```
side x side      5  12   29  ...
area octagon   24  58  140  ...
50,625 x 29 x '140  =  10,487 (rounded)
    101 x 101  =  10,201  ---  286 less
    102 x 102  =  10,404  ---   83 less
    103 x 103  =  10,609  ---  122 more
```

The side length of the octagon lies between 102 and 103 royal cubits, nearer to 102 cubits. By multiplying these numbers by a factor of 5 we obtain:

```
  '2 x 225 x 5 x 450 x 5  =  1,265,625
1,265,625 x 29 x '140  =  262,165 (rounded)
     511 x 511  =  261,121  ---  1044 less
     512 x 512  =  262,144  ---    21 less
     513 x 513  =  263,169  ---  1004 more
```

We have found a good value: the side of the octagon measures practically 512 x '5 cubits. Now for the grid:

```
3 x 140  plus  58  plus  17  plus  17  =  512
3 x  99  plus  41  plus  12  plus  12  =  362
5 x 99  plus  17  =  512
5 x 70  plus  12  =  362
```

Hence the grid measures

```
362+512+362 by 362+512+362  fifth royal cubits
```

while the area of the octagon in the grid is found as follows:

```
362+512+362  =  1236 fifth royal cubits
1236 x 1236  -  2 x 362 x 362  =  1,265,608
1,265,608 x '5 x '5  =  50,624 '5 '10 '50 square cubits
      area of triangle  =  50,625 square cubits
        mistake only  '2 '6 '75 square cubit
```

We have been lucky: we found a result with a tiny margin of error of less than 1 square cubit in an area of over 50,000 square cubits. Or the other way round: the numbers of the triangle were chosen because they would yield a fine result.
Now for the remaining calculation of RMP 53 (translation by Thomas Eric Peet, measurement transformed by myself):

```
1/10 of an area measures 14,750 square cubits
1/10 of the area subtracted, then this is the area
```

There are two areas, one of them '10 smaller than the other - perhaps an octagon in a circle? Let us consider an octagon in a simple grid:

```
 side octagon  10   square  24 by 24
 partition  7+10+7 = 24   grid  7+10+7 by 7+10+7
 diameter of the inscribed circle  24
 diameter of the circumscribed circle  26
  according to the triple  2 x 5-12-13
 diameter 26 / side 10  =  13 to 5
```

The area of the octagon measures

```
 24x24 - 2x7x7  =  478 square units
```

The area of the circle is given by the following formula:

radius x radius x re

The radius measures 13 units:

```
 13 x 13 x re
```

Is there a good value for re? Yes, 531 above 169:

```
 3  (plus 22)  22  47  69  91  113  ...  531
 1  (plus  7)   8  15  22  29   36  ...  169 = 13x13
 13 x 13 x 531 x '13 x '13  =  531
 Area of the circle                531 square units
 area of the inscribed octagon     478 square units
```

The ratio is practically 10 to 9, according to a crosswise multiplication:

```
  531 x 9 = 4779    478 x 10 = 4780
```

Hence the two areas mentioned in RMP 53 may really be a circle and an octagon.
Now let me invent a problem: The radius of a circle measure 2 '6 khet or '3 x 650 royal cubits - how long is the side of the inscribed octagon? I calculate the area of the circle by means of the formula:

```
  radius  x  radius  x '169 x 531
 '3 x 650 x '3 x 650 x '169 x 531  =  147,500 square cubits
 area of the circle                   147,500 square cubits
 minus one tenth (RMP 53)              14,750 square cubits
 = area of the inscribed octagon      132,750 square cubits
 Diameter / side  =  13 to 5
 diameter  =  2 x 2 '6 khet  =  4 '3 khet  =  '3 x 13 khet
 side  =  '3 x 13 x 5 x '13 khet  =  '3 x 5  or  1 '6 khet
```

If the radius of a circle measures 2 '6 khet, the side of the inscribed regular octagon measures about 1 '6 khet.
A more precise result is derived by means of another octagon. Draw the square 24 by 24: the diagonals measure practically 34 units. Prolong the axes by 5 units on every side: they will measure 34 units. Join the ends of the axes and the corners of the square, and you obtain an octagon: its side measures 13 units, according to the triple 5-12-13

```
 Diameter / side  =  34 to 13
```

The diameter of my circle measures

```
4 '3 khet or '3 x 1300 royal cubits
```

while the side of the inscribed octagon measures

```
'3 x 1300 x 13 x '34  =  165 '2 '6 '51 royal cubits
margin of error: only 1 palm on 165 royal cubits
```

This is my interpretation of RMP 53 so far. It would no doubt have provided subject matter for a whole semester in the seminary of professor Ahmes.

RMP 54 and 55

7 setat of land are divided into 10 fields, each one measuring 7,000 square cubits.
3 setat of land are divided into 5 fields, each one measuring 6,000 square cubits.
You may say that these are very simple calculations compared to RMP 53. But Ahmes will smile, and propose that you transform these areas into regular octagons. You will try this, and find the following solutions:

```
OCTAGON A
side  38 royal cubits
square  92 by 92 royal cubits
partition  27 + 38 + 27  =  92 royal cubits
grid  27+38+27 by 27+38+27  royal cubits
area octagon (grid)  7,006 square cubits
OCTAGON B
side  35 royal cubits
square  85 by 85 royal cubits
grid  25+35+25 by 25+35+25 royal cubits
area octagon (grid)  5,975 square cubits
```

Ahmes will be pleased. Then he may propose that you multiply the numbers by a factor of 20, refine the grids and look out for a pair of new octagons. With your previous learning you will find the following solutions quite easily:

```
OCTAGON A
side  704   (20 units = 1 royal cubit)
square  1700 x 1700  (85 by 85 royal cubits)
partition  498 + 704 + 498  =  1700  (85 royal cubits)
grid  498+704+498 x 498+704+498
OCTAGON B
side  762   (20 units = 1 royal cubit)
square  1840 x 1840  (92 by 92 royal cubits)
partition  539 + 762 + 539  =  1840  (92 royal cubits)
grid  539+762+539 x 539+762+539
```

Then Ahmes will ask you to draw the grids and octagons, using Maantef marks instead of royal cubits (20 Maantef marks equal 10 Nut marks or 1 finger or about 1.87 centimeters):

```
OCTAGON A and B
partitions  539+762+539  and  498+704+498 Maantef marks
```

Next, Ahmes will ask you to draw a circle around the smaller octagon and another one inside of the larger octagon. By doing so you will be surprised: the circles have the same diameter:

```
OCTAGON A    side  762 Maantef marks
square  1840 by 1840 Maantef marks (92 by 92 fingers)

grid  539+762+539 by 539+762+539 Maantef marks
diameter of the inscribed circle  1840 Mm (92 fingers)
OCTAGON B    side  704 Maantef marks
square  1700 by 1700 Maantef marks (85 by 85 fingers)
grid  498+704+498 by 498+704+498 Mantef marks
diameter of the circumscribed circle  1840 Mm (92 fingers)
 according to the pseudo-triple  4 x 176-425-460
```

RMP 56

The base of a pyramid measures 360 royal cubits and its height 250 royal cubits. Beginners calculate the sekad: how much does the slope recede on 1 royal cubit of height:

```
half base / height  =  180 : 250  =  '2 '5 '50
sekad  1 royal cubit x '2 '5 '50  =  5 '25 palms
```

Advanced learners may solve more demanding problems:
a) imagine a circle in the square of the base and calculate the circumference
b) transform the base into a regular octagon of about the same area; imagine a circle
in the square of the grid and calculate the circumference of that circle
c) transform the volume of the pyramid into a cube
Let us again consult the following number sequence:

```
3  (plus 22)  25  47  69  ... 289  311  333  355  377
1  (plus  7)   8  15  22  ...  92   99  106  113  120
```

The diameter of our circle measures 360 = 3 x 120 royal cubits. How long is the circumference? 3 x 377 = 1131 royal cubits. The area of the base measures 360 x 360 = 129,600 square cubits. Octagon of about the same area:

```
side  164 royal cubits
grid  116+164+116 by 116+164+116 royal cubits
area (grid)  396x396 - 2x116x116  =  129,904 square cubits
side length of grid  396 rc   diagonal  560 royal cubits
diameter of inscribed circle  4 x  99  =   396 royal cubits
circumference                 4 x 311  =  1244 royal cubits
```

Finally, the cube. The volume of the pyramid measures

```
 360 x 360 x 250 x '3  =  60 x 60 x 60 x 50 cubic cubits
```

while the edge of a cube of the same volume measures

```
 60 royal cubits x cube root of 50
```

How do we approximate the cube root of 50? By using the equation

```
 A x A x A  =  50 x B x B x B  plus/minus C
```

and looking out for numbers A and B that will keep C small:

```
  4 x  4 x  4  =  50 x  1 x  1 x  1   plus 16
 11 x 11 x 11  =  50 x  3 x  3 x  3   minus 19
 70 x 70 x 70  =  50 x 19 x 19 x 19   plus 50
```

The value 4 is too great: `4 x 4 x 4  =  64`

The value '3 x 11 is a little too small:

```
 '3 x 11 x '3 x 11 x '3 x 11  =  '27 x 1331  =  49 plus ...
```

The value '19 x 70 is again a little too great:

```
 '19 x 70 x '19 x 70 x '19 x 70  =  50 plus ...
```

Pairs of such values generate further and better values:

```
 4  (plus 11)  15  26  37  48  59  70  81
 1  (plus  3)   4   7  10  13  16  19  22
 11  (plus 70)  81  151  221  291  361  431  501  571
  3  (plus 19)  22   41   60   79   98  117  136  155
 641  711  781  851  921  991  1061  1131  1201
 174  193  212  231  250  269   288   307   326
```

The value '307 x 1131 is the best approximation for the cube root of 50 (funny, we have already seen the number 1131), while the numbers 221 and 60 yield a simple solution to our problem:

```
 60 x cube root of 50  =  about  60 x 221 '60  =  221
```

A pyramid with a base of 360 royal cubits and a height of 250 royal cubits and a cube of the edge 221 royal cubits have nearly the same volume.

RMP 57 and 58

The base of a pyramid measures 140 royal cubits, the height 93 '3 royal cubits, and the sekad 5 palms 1 finger. Advanced learners may solve the following problems: Imagine a circle in the square of the base and another around the base: how long are the circumferences? Transform the area of the base into a circle: how long are the slopes of the pyramid? Imagine a hemisphere and a sphere in the frame of this pyramid: how long are the radii?

The base measures 140 royal cubits, the diagonal 198 royal cubits, and the average 169 royal cubits. If we regard these numbers as diameters of 3 circles, the circumferences measure:

```
140 royal cubits  x   '7 x 22   =  440 royal cubits
198 royal cubits  x  '99 x 311  =  622 royal cubits
169 royal cubits  x '169 x 531  =  531 royal cubits
```

How can we transform the area of the base into a circle? We may use the value '7 of 22 and work with the following equation, looking out for good values of A and B which will keep C small:¨

```
22 x A x A  =  7 x B x B  plus minus C
22 x 22 x 22  =  7 x 39 x 39  plus 1
```

The numbers 22 and 39 provide a fine approximation for the square root of 're'. The pyramid base measures 140 royal cubits. I multiply this number by 22 and obtain 3080. Now I divide 3080 by 39 and obtain 79 minus '39 or practically 79. Hence, a circle of the radius 79 royal cubits and a square of the side 140 royal cubits have about the same area.

Now for the slope. The height measures '3 x 280 royal cubits, half the base measures '3 x 210 royal cubits, and the slope measures exactly '3 x 350 or 116 ''3 royal cubits - according to the Sacred Triangle 3x70-4x70-5x70 = 210-280-350.

Now for the final answers: the radius of the imaginary sphere in the frame of this pyramid measures exactly 35 royal cubits, while the radius of the inscribed hemisphere measures exactly 56 royal cubits.

The imaginary sphere and hemisphere in the frame of the pyramid symbolize the sun and the sky enclosed in the Primeval Hill.

RMP 59

Base and height of a pyramid measure 12 and 8 royal cubits. Let us imagine a wooden model of this pyramid:

```
height  8 fingers   base 12 fingers
half base  6 fingers  or  3 x 2 fingers
height     8 fingers  or  4 x 2 fingers
slope     10 fingers  or  5 x 2 fingers
```

This pyramid is again defined by a Sacred Triangle, so let me call this type of a pyramid a 'Sacred Pyramid'.

The surface of the model (base and four faces) measures 384 square fingers. How much is the volume? 384 cubic fingers.

The radius of the inscribed sphere measures exactly 3 fingers, while the distance from the center of the sphere to a corner of the base measures exactly 9 fingers (quadruple 3-6-6-9).

Now let us calculate the volume of a sphere of the diameter 9 fingers. Using the value '81 x 256 for 're' we again obtain 384 cubic fingers. This generates a pretty formula: A square of the side 8 units and a circle with a diameter of 9 units have nearly the same area; a Sacred Pyramid with a height of 8 units and a sphere with a diameter of 9 units have nearly the same volume

RMP 60

Imagine a cone. The diameter of the base measures 15 royal cubits, and its height measures 30 royal cubits. Calculate the volume of the cone. Then imagine a sphere of the same diameter, 15 royal cubits. Calculate the volume of the sphere, using the same value for *re* (e.g. '15 of 47). The cone and the sphere have exactly the same volume.