

River Map Ho-T'u
and Magic Square Lo-shu
of Early China

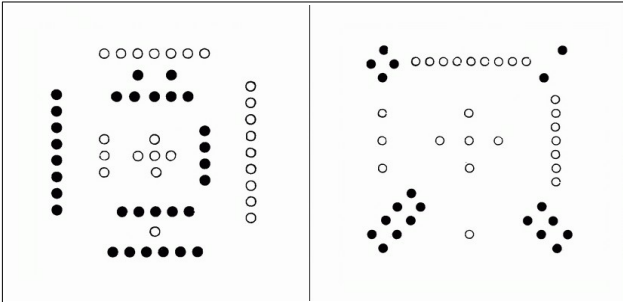
Franz Gnaedinger Zurich © 2012

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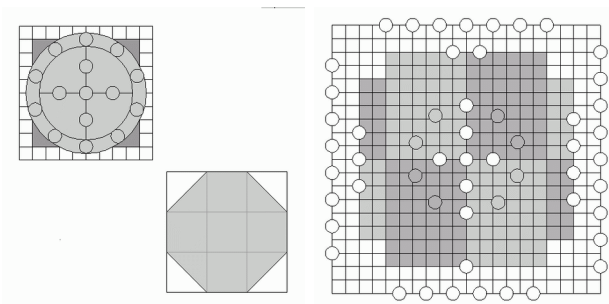
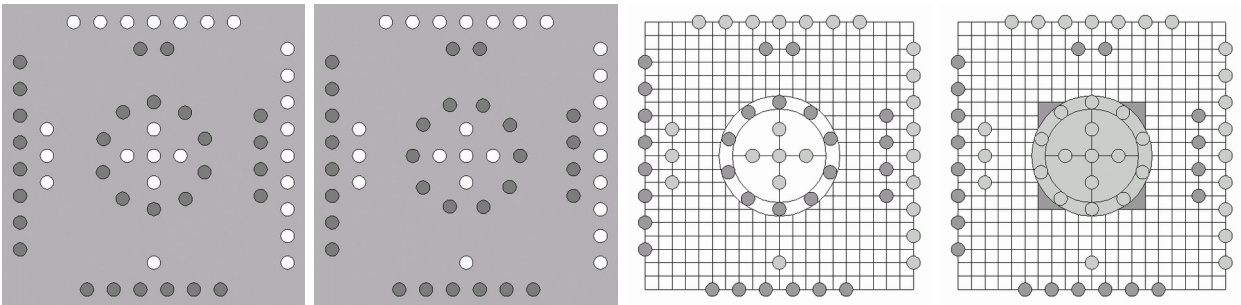
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Illustrations

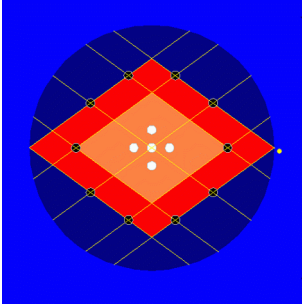
River Map and Magic Square, from: James Legge, *The I Ching*, Clarendon Press 1899, Dover Publications New York 1963 (a very fine book that inspired this paper)



The same square and practically the same map are given by David Eugene Smith in *A History of Mathematics*, 1923, Dover reprint 1953/58 (richly illustrated, one of my favorite sources), only that the five and five dark circles of the earth below and above the heavenly domino five are, for the sake of clarity, connected with a rectangle. A modified reconstruction of the lost diagram is proposed in this paper



The circle might keep a memory of a Neolithic calendar sanctuary, a circle of poles providing sighting lines for the rising and setting sun on the solstices (and equinoxes), on the right side the morning sun of the spring equinox that just rose over the horizon, somewhere on the northern part of the Ordos Plateau, a couple of millennia ago



Part 1. Maps

Mapping the River of Time – a lunisolar calendar encoded in the Yellow River Map of ancient China that inspired the I Ching or Book of Changes

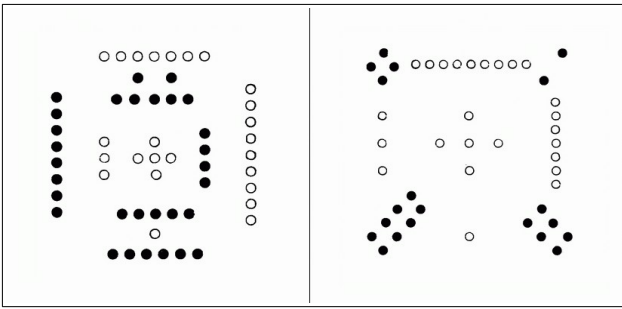
The Stone Age way of counting lunations was to lay out pebbles, 30 29 30 29 30 29 30 ... pebbles for 1 2 3 4 5 6 7 ... lunations or synodic months.

15 and 17 lunations counted that way yield 443 and 502 days respectively, together 945 days for 32 lunations, doubled 1890 days for 64 lunations.

The Göbekli Tepe lunisolar calendar had a month of 30 days and a basic year of 12 months or 360 days. Add 3 days of midsummer and 2 days of midwinter and you get a regular year of 365 days. Add 3 midwinter days instead and you get an occasional leap year of 366 days.

63 periods of 30 days are 1890 days and correspond to 64 lunations, as found above. One lunation, calculated that way, lasts 29 days 12 hours 45 minutes, exact value 29 d 12 h 44 m 2.9 s (average from 1989 AD), mistake less than one minute per lunation, or half a day in a lifetime.

Now for the River Map Ho-T'u. A dragon-horse emerged from the Yellow River Ho, bearing a map on its back, the Yang numbers 1 3 5 7 9 given in light circles and the Yin numbers 2 4 6 8 10 given in dark circles arranged in a square, inner square 1 3 2 4 sum 10, outer square 6 8 7 9 sum 30, individual numbers given as lines of light or dark circles, beginning with 1 at the bottom of the inner square, in clockwise direction. The center is held by a domino five in light circles, below and above a line of five dark circles each, together the Yin number 10.



The number 10 represents a basic year of ten periods of 36 days, in all 360 days. Add the vertical line of the domino five for 3 midsummer days and the remaining circles for 2 midwinter days and you get a regular year of 365 days. If you add instead the horizontal line of the domino five for 3 midwinter days you obtain an occasional leap year of 366 days.

25 years require 6 leap years (in all 9131 days). The number 6 is given as sum of the vertical line and horizontal line of the domino five (central circle counted twice). The number 25 is the sum of the Yang numbers 1 3 5 7 9.

The number 36 of the calendar period is the sum of the numbers 1 2 3 4 5 6 7 8. The square of 8 equals 64. Outer square of the River Map 6 8 7 9 sum 30. The product of the Yang numbers 7 and 9 equals 63, times 30 days 1890 days or 64 lunations or 2 by 2 by 2 by 2 by 2 by 2 lunations, mirrored in the cycle of 64 binary hexagrams in the I Ching.

64 lunations correspond to 63 periods of 30 days, 128 lunations to 126 periods of 30 days or 105 periods of 36 days, yielding the l/p ratio 126/105. Simpler ratios are 6/5 or 12/10 and 11/9. Generating additive number sequences of more l/p ratios:

6/5 (plus 11/9) 17/14 28/23 39/32 50/41 61/50 ...

Best value: 50 periods of 36 days for 61 lunations.

12/10 (plus 11/9) 23/19 ... 78/64 89/73 ...

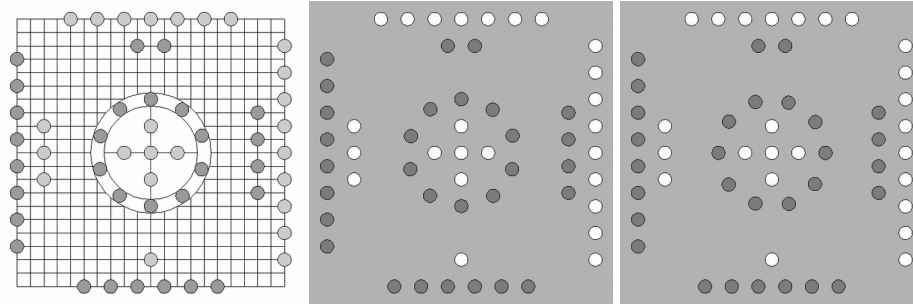
Best value: 73 periods of 36 days for 89 lunations.

89/73 (plus 128/105) 217/178 345/283 473/388 ...

Best value: 388 periods of 36 days for 473 lunations, mistake not even six seconds per lunation, a good hour in a lifetime.

The period of 36 days offers many practical, good, very good and excellent values for counting lunations over long periods of time – mapping the river of time, as it were.

The River Map is a reconstruction. I'd like to make a suggestion: instead of halving the Yin number 10 one might arrange it as a circle around the domino five. Outer square 20 by 20, inner square 16 by 16, diameter of a small circle 1, distance between the centers of two small circles on a straight line 2, radius of the circle of the basic year 4, diameter 8, outer diameter 9, inner diameter 7:



The River Map Ho-T'u was complemented by the Magic Square Lo-Shu on the back of a tortoise that also emerged from the Yellow River Ho (David Eugene Smith)

4 9 2
 3 5 7
 8 1 6

- 5 heaven (domino five)
- 1 north and winter and water
- 3 east and spring and wood
- 9 south and summer and fire
- 7 west and autumn and metal
- 2 earth
- 4 wood
- 6 metal
- 8 earth

The Magic Square may be seen as a world map. Time is present insofar as the cardinal directions go along with the seasons of the year.

Part 2. River of Time and Change

Where is the river in the Yellow River Map?

I explained how the Yin number 10 can be arranged as a circle around the domino five in the center of the map. Now imagine that circle in motion, turning in clockwise direction, round and round, making the seasons change, from winter to spring to summer to autumn, from cold to warm to hot to cool, bringing about changes of every sort – as the waters in a river rise and sink and rise in perpetual motion.

The number 10 of the river circle is also the sum of the pairs opposing each other via the heavenly domino five in the Magic Square

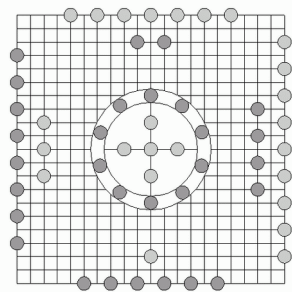
4 9 2
 3 5 7
 8 1 6

1 and 9, north and south, winter and summer, water and fire; 3 and 7 east and west, spring and autumn, wood and metal; 4 and 6, wood and metal; 2 and 8, earth. East and west also invoke

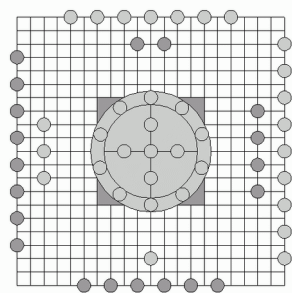
morning and evening. Wood and metal are organic and anorganic materials. Earth makes the plants grow and in turn takes up the fallen leaves. Consider also the colors, the yellow of dawn and violet of dusk, blue water and orange fire, green leaves and rusty red iron ore, turning into a glowing red when molten with fire made of wood ... The elements of the Magic Square, paired in opposites, evoke a host of meanings.

We could say that the eight numbers around the heavenly domino five in the center of the Magic Square are the frozen River of Time and Change that flows around the domino five in the center of the River Map.

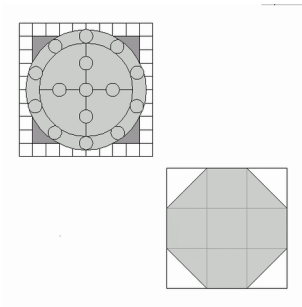
Here again the numbers of the modified reconstruction of the Ho Map: outer square 20 by 20, inner square 16 by 16, diameter of a small circle 1, distance of the centers of two adjacent small circles on a straight line 2, radius of the river circle 4, diameter 8. Those are middle numbers, relevant for the positions of the centers of the small circles. 8 is the middle diameter of the river circle. The outer diameter is 9, the inner diameter 7.



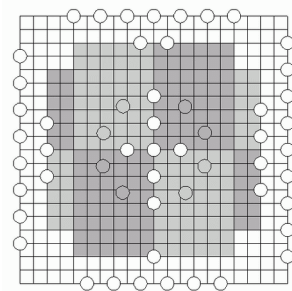
7 8 9 play a role in practical methods of squaring the circle in the Rhind Mathematical Papyrus of Ancient Egypt. A circle of diameter 9 and a square of side 8 have practically the same area.



A small drawing shows a square with a virtual or imaginary grid of 3 by 3 squares whose corner squares are halved along diagonals, generating a quasi-circle, an octagon that is close to the circle inscribed in the square 3 by 3. Also the numbers 1 2 3 4 5 6 7 8 9 of the Magic Square fill a grid of 3 by 3 squares. Their sum is 45. Halve the numbers of the corner squares 2 4 6 8 and the sum is reduced to 35. The ratio 45 to 35 equals 9 to 7.



The Egyptians of the Old Kingdom developed a systematic method for the calculation of the circle on the basis of the Sacred Triangle 3-4-5. The numbers of this triangle are prominent in the River Map: 3 on the left side of the inner square, 4 on the right side of the inner square, 5 in the center.



The first Egyptian circle of radius 5 in the square 10 by 10 is defined by the triple 3-4-5 that marks 8 rational points of the circumference. The second circle of radius 25 is defined by the triple 15-20-25, enlarged version of the basic triple 3-4-5, and the new triple 7-24-25 that marks 8 more points of the circumference. And so on. Every multiplication by a factor of 5 yields one more triple that marks 8 more points on the circumference. Now the numbers 5 and 8 and 10 are present in the River Map and the Magic Square. The number 25 is given by the sum of the Yang numbers of the cross 1 3 5 7 9, and again by the oblique cross of the Yin numbers 2 4 6 8 around the heavenly 5. Add the numbers 25 of the one cross and the number 25 of the other cross and you get 50.

While the calendar concerns time, the squaring of the circle concerns a change of form, a shift of shape, justifying the ‘River of Time and Change’ that could also have named the circle of hexagrams in the I Ching. There is a rational core in the Chinese instrument of divination.

Part 3. Unfolding Cosmos

Where did the Yang line and Yin line come from? A Chinese author by the name of Ku Shi said their origin was a circle. The circle divided and produced the whole line of the Yang and the divided line of the Yin, which lines, in turn, produced all things ... The initial circle may then have been the empty circle Wu of the primeval cosmos before the division into the Yang of heaven and Yin of earth.

Let me imagine a myth of creation as it might have been told in the Paleolithic or Mesolithic or Neolithic settlements on the Ordos Plateau in the wide northern curve of the Yellow River.

In the begin was the empty circle Wu. Then the circle divided. The upper half became the Yang, manifest in heaven, and the lower half became Yin, manifest in earth. Hereupon the Tree of Life grew out of the earth, marking the center of the world – here, in the center of the earth, under the

center of heaven –, and dividing the world into north and south, east and west, appearing as a cross or domino five when imagined from above, and as a pair of lines when imagined from the side: a whole upper line, and a divided lower line, the center open for the Tree of Life.

Yin and Yang produced all things, and when we play with the numbers and forms of the (modified) River Map and the Magic Square we see a mathematical cosmos unfold.

1 1 2, 2 3 4, 5 7 10, 12 17 24, 29 41 58, 70 99 140 ... these are numbers of the square and octagon.

1 1 3, 2 4 6, 1 2 3, 3 5 9, 8 14 24, 4 7 12, 11 19 33, 30 52 90, 15 26 45 ... these are numbers of the equilateral triangle, hexagon, and cube.

1 1 5, 2 6 10, 1 3 5, 4 8 20, 2 4 10, 1 2 5, 3 7 15, 10 22 50, 5 11 25, 16 36 80, 8 18 40, 4 9 20 ... these are numbers of the double square.

3 4 5, the numbers of the Sacred Triangle, prominent in the River Map, start a sequence of ever rounder polygons whose peripheries can be calculated with the numbers of the square and double square.

4 1 1 are the numbers of the periphery, the horizontal and vertical axis of the unit square. 3 1 1 are the numbers of the simplified circumference, horizontal and vertical diameter of the unit circle, reflected in the Yang number 9 and Yin number 3 3, also in the Yang number 36 and Yin number 12 12 of the I Ching. The 64 hexagrams count 192 Yang lines and 192 Yin lines, yielding 11,520, “the number of all things” (I Ching, Appendix III 53).

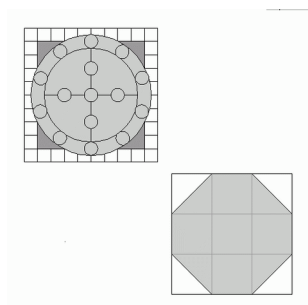
4 1 are the periphery and axis of the unit square, 3 1 the simplified circumference and diameter of the unit circle. Begin with 4/1 and add repeatedly 3 to 4, and 1 to 1:

4/1 (plus 3/1) 7/2 10/3 13/4 16/5 19/6 22/7 25/8 28/9

3/1 (plus 22/7) 25/8 ... 311/99 ... 377/120

9/3 (plus 19/6) 28/9 ... 256/81

The grid 3 x 3 has the area 9. If you halve the corner squares along diagonals, in such a way that you get an octagon, you have a good first approximation of the circle inscribed in the square: area of the quasi-circle 7, implicit pi-value 28/9. A circle of diameter 9 and a square of side 8 have practically the same area, implicit pi-value 256/81.



2 3 2 sum 7 – make a rod of that length. 4 3 4 sum 11 – make a second rod of this length. If the diameter of a circle measures one short rod, the circumference measures two long rods. If the radius of another circle measures one short rod, the area equals two short rods by one long rod. If the side of a square measures ten short rods, the diagonal measures 9 long rods, and if the side of a square

measures nine long rods, the diagonal measures twenty short ones. Three short rods are the golden minor of five long rods, in numbers 21 and 55, from the Fibonacci sequence 1 1 2 3 5 8 13 21 34 55 89 144 ..., a sequence present in the above numbers of the double square, as are the complementary Lucas numbers 1 3 4 7 11 18 29 47 76 123 199 322 ...

11,520 is the number of all things according to Appendix III 53 of the I Ching. Why that specific number? One reason was given above, the other reason is a challenging problem of an amazingly simple answer. Imagine a circle of the circumference 11,520. How long is the periphery of the square of the same area? 13,000. Implicit value for the square root of pi 576/325, an excellent value from the sequence

16/10 (plus 16/9) 32/19 48/28 ... 576/325 592/334 (296/167)

We observe a mathematical cosmos unfold from the River Map Ho-T'u and the Magic Square Lo-Shu and evolve in time, on the way to the I Ching and the important third appendix.

Postscript

A Neolithic calendar sanctuary on the Ordos Plateau?

The central part of the modified River Map could have served as a Neolithic calendar sanctuary on the northern (northwestern) part of the Ordos Plateau (geographical latitude around 40 degrees, perhaps nine thousand years ago). A circle of ten poles would have allowed to predict sunrises and sunsets on the solstices (and equinoxes), a flat horizon and a numerical model of the year provided. The rhomb is given by the sighting lines of the solstices. (Numbers of the drawing: middle radius 34, middle diameter 68, outer diameter 72, inner diameter 64, diameter pole 4, long axis $2 \times 55 = 110$, short axis $2 \times 40 = 80$, side of rhomb 13 21 21 13 sum 68.) The morning sun of the spring equinox just rose over the eastern horizon (tiny yellow circle on the right side)

