Babylonian Mathematics

All IS EQUAL, ALL UNEQUAL …

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Babylonian clay tablet YBC 7289

On the Babylonian clay tablet YBC 7289 is drawn a square, with the side measurement indicated as 30, while the numbers 42;25,35 and 1;24,51,10 are written on its diagonal. If the side of a square measures 30 units, the diagonal measures practically 42;25,35 units, while the corresponding value for the square root of 2 equals 1;24,51,10 in the sexagesimal number system of Babylon. This excellent value can be found by means of the following number pattern:

```
1       1       2
  2       3       4
    5       7      10
     12      17      24
       29      41      58
         70      99     140
           169     239     338
             408     577     816
               985    1393    ....
```

Divide 1393 by 985 and you obtain 1;24,51,10,3,2... Let the small numbers go and keep 1;24,51,10.- Imagine a circle inscribed in the square 30 by 30. Calculate the circumference. Using a pocket calculator you will find an amazingly good approximation:

diameter 30   circumference 1,34;14,52

This number is based on an excellent value for pi: 84823 / 27000 = 3;8,29,44

This value is even better than 355/113. Could the Babylonians possibly have found such a fine value? Yes, they could easily have done so by using the number sequences of their Egyptian colleagues:

```
4   (plus 3)   7  10  13  16  19  22  25
1   (plus 1)   2   3   4   5   6   7   8
3   (plus 22)  25  47  69  ...  311  333  355  377
1   (plus  7)   8  15  22  ...   99  106  113  120
333  (plus 355)  688  1043  ...  2463          ...  84823
106  (plus 113)  219   332  ...   784 = 28x28  ...  27000
```

Let a cube measure 30 by 30 by 30 units. The volume equals

30 x 30 x 30 = 27000 cubic units

while the squared cubic diagonal equals

30x30 plus 30x30 plus 30x30 = 2700 square units

Imagine the cube holding a sphere and calculate the volume of the sphere. It would simply measure 84823/6 cubic units. Imagine a sphere holding the cube and calculate the surface of the sphere. It would simply measure 84823/10 square units. Both results can easily be given in the sexagesimal form.

Plimpton 322

Papyrus was a precious and highly expensive material, and there is little writing space on a Babylonian clay tablet. These limitations may have led the Egyptian and Babylonian mathematicians of the second millenium BC to convey their knowledge by way of telling examples. I dare also say that the 15 triples mentioned in the famous Babylonian clay tablet Plimpton 322 are a 'book' of mathematics in a very condensed form. Let me have a look at the 15 triples and invent some tasks to go with the numbers. First a few Sumerian measures:

1 gish (about 360 m) = 6 es (rope, 60m) = 60 gar (6m) = 120 gi (cane, 3 m) = 720 kush (cubit, 50 cm) = 1440 shubad (span, 25 cm) = 21600 shusi (finger, 1.7 cm)

First triple (120) - 119 - 169 (kush)

The sides of a rectangular triangle measure 119-120-169 kush. How long is the diameter of the inscribed circle? Simply

119 + 120 - 169 = 70 kush

The circumference measures practically 220 kush.

```
4  (plus 3)  7  10  13  16  19  22
1  (plus 1)  2   3   4   5   6   7
3  (plus 22)  25  ...  377  ...  531
1  (plus  7)   8  ...  120  ...  169
```

Let the side of a square measure 119 = 7 x 17 kush. The diagonal measures practically.7 x 24 = 168 kush. The circumference of the inscribed circle measures practically 22 x 17 = 374 kush.

Let the side of a square measure 1 es = 120 kush. The diagonal measures practically 170 kush. Calculate the circumference of the inscribed circle using the Babylonian pi-values 3 and 25/8, the better value 377/120, and the excellent value 1;8,29,44 found via YBC 7289. You will obtain 3 es, 3 es 15 kush, 3 es 17 kush, and 3 es 16 kush 29;44 shusi respectively.

Let the side of a square measure 169 kush. The circumference of the inscribed circle measures practically 531 kush, the diagonal of the square practically 119 + 120 = 239 kush:

```
1       1       2
  2       3       4
    5       7      10
      12      17      24
        29      41      58
          70      99     140
            169     239     338
              408     577     816
                985    1393    ....
```

The lines of this basic number pattern contain the triples and quadruples ( )-1-1, 1-2-2-3, 3-4-5, 8-9-12-17, 19-20-21, 49-50-70-99, 119-120-169, 288-289-408-577 ...

Second triple (3456) - 3367 - 4825 (kush)

Let a rectangle measure 3367 by 3456 kush or 6734 by 6912 shubad. Square the circumscribed circle. The diagonal of the rectangle measures 4825 kush, the radius of the circle 4825 shubad, and the side of the square of the same area practically 8552 shubad or 4276 kush, according to the following number sequences for pi and the square root of pi:

```
4  (plus 3)  7  10  13  16  19  22  25  28  ..  49 = 7x7
1  (plus 1)  2   3   4   5   6   7   8   9  ..  16 = 4x4
9  (plus 19)  28  47  66  ...  256 = 16x16
3  (plus  6)   9  15  21  ...   81 =  9x9
3  (plus 22)  25  47  69  ...  1521 = 39x39
1  (plus  7)   8  15  22  ...   484 = 22x22
1  (plus 16)  17  33  49  ...  177  ...  225  241  257
1  (plus  9)  10  19  28  ...  100  ...  127  136  145
7  (plus 39)  46  85  124  ...  475  514  553  592
4  (plus 22)  26  48   70  ...  268  290  312  334
257  (plus 553)  810  1363  1916  2469  ....  8552
145  (plus 312)  457   769  1081  1393  ....  4825
```

Two neighboring values in a sequence above generate a new sequence that contains a still better and easily convertible value:

```
553  (plus 592)  1145  1737  ....  7065  7657
312  (plus 334)   646   980  ....  3986  4320
7657 / 4320  =  382850 / 216000  =  1;46,20,50
```

If you wish to square a circle you may multiply the radius by 1;46,20,50. Thus you obtain the side of a square of practically the same area.

Third triple (4800) - 4601 - 6649 (kush)

Let a rectangle measure 4601 by 4800 kush. The diagonal of the square of the same area measures practically 6646 kush (square root of 2 x 4601 x 4800). The circumference of the circumscribed circle measures practically 20879 kush (29 gish minus 1 kush).

```
3 (plus 22) ... 289 ... 355   289 (plus 355) ... 20879
1 (plus  7) ...  92 ... 113    92 (plus 113) ...  6646
```

Fourth triple (13500) - 12709 - 18541 (kush)

Turn a triangle of these numbers in a square of the same area. The side measures practically 9262 kush. Now turn the rectangle 13500 by 12709 kush in a circle of the same area. Multiply 9262 kush by 1;24,51,10 and divide the result by 1;46,20,50. Thus you will obtain the radius. It measures practically 7390 kush.

Fifth triple (72) - 65 - 97 (kush)

The diagonal of the rectangle 72 by 65 kush measures 97 kush or 97 x 30 = 2910 Sumerian fingers. The circumference of the circumscribed circle measures practically 9142 fingers.

```
 3 (plus 22) ... 311 ... 355   311 (plus 355) ... 4571 (9142)
 1 (plus  7) ...  99 ... 113    99 (plus 113) ... 1455 (2910)
```

Sixth triple or triangle (360) - 319 - 481 (kush)

Imagine a rectangular triangle with these measurements. The diameter of the inscribed circle measures 319 + 360 - 481 = 198 kush, and the circumference practically 622 kush (pi value 311/99).

Seventh triangle (2700) - 2291 - 3541 (kush)

The diameter of the inscribed circle measures 1450 kush. Square the circle. The side of a square of the same area measures practically 1285 kush.

Eighth triangle (960) - 799 - 1249 (kush)

The diameter of the inscribed circle is 510 kush. The side of the squared circle measures practically 452 kush.

Ninth triangle (600) - 481 - 769 (kush)

diameter inscribed circle 312 kush, circumference 980 kush

```
 3  (plus 22)  25  47  69  ...  245 (980)
 1  (plus  7)   8  15  22  ...   78 (312)
```

The area of the circle is practically 76,440 square kush, the diagonal of the squared circle practically 391 kush.

Tenth triangle (6480) - 4961 - 8161 (kush)

Use the periphery 4 x 99 x 99 shubad as diameter of a circle. The circumference measures practically 123,163 shubad:

```
 333  (plus 355)  ...  2463   ....  84823  ....  123163
 106  (plus 113)  ...  28x28  ....  27000  ....  4x99x99
```

Eleventh triple (60) - 45 - 75 (kush)

Let a rectangle measure 60 by 45 kush. The circumscribed circle also holds the rectangle 72 by 21 kush. Basic triples 3-4-5 and 7-24-25. Let a grid measure 150 by 150 kush. The inscribed circle passes 20 points of the grid. Draw a circle around a building using the grid 150 by 150 and the triples 45-60-75 and 21-72-75 kush.

Twelfth triple (2400) - 1679 - 2929 (kush)

```
 alternative triple 2020-2121-2929, basic triple 20-21-29
 alternative triple  580-2871-2929, basic triple 20-99-101
```

Thirteenth triple (240) - 161 - 289 (kush)

alternative triple 136-255-289, basic triple 8-15-17

The Egyptian method for calculating pi works with every starting triple, not only with the Sacred Triangle 3-4-5 but also for example with the triple 8-15-17:

```
starting triple a-b-c = 8-15-17
+- 8a +- 15b  (choose the positive value not divisible by 17)
+- 8b +- 15a  (choose the positive value not divisible by 17)
    17c
 8-15-17   136-240-289   2312-4335-4915   .....
      161-240-289   2737-4080-4915   .....
             495-4888-4915   .....
```

Calculate the first polygon using the ratios 10/7 and 25/6 for the square roots of 2 and 17, and the second one using the ratios 17/12, 25/6 and 35/6 for the square roots of 2, 17 and 34. You will obtain 160/51 and 5443/1734 for pi.

```
6 (plus 22) ... 160      9 (plus 22) ... 5443
2 (plus  7) ...  51      6 (plus  7) ... 1734
```

Fourteenth triangle (2700) - 1771 - 3229 (kush)

The periphery measures 7700 kush. The diameter of a circle of the same circumference measures about 2450 kush (pi value 22/7), or more precisely 2451 kush:

```
245  (plus 355)  600  955  1310  ....  7345  7700
 78  (plus 113)  191  304   417  ....  2338  2451
```

Fifteenth triple (90) - 56 - 106 (kush)

Let a rectangle measure 90 by 56 kush. The diagonal measures 106 kush. The circumference of the circumscribed circle measures practically 333 kush. Turn the rectangle in a square of the same area. The side of the square and the circumference of the inscribed circle measure practically 71 and 223 kush:

```
3  (plus 22)  25  47  69  ...  223  ...  333
1  (plus  7)   8  15  22  ...   71  ...  106
```

PS from November 2002

The number patterns for the calculation of the square are generators of triples:

```
1     1     2          2     7     4
 2     3     4          9    11    18
  5     7    10         20    29    40
   12    17    24         49    69    98
     29    41    58         118   167   236
       70    99   140          ...
        169   239   338
          408   577   ...
1   1   2      2x2  1x3  1x1+2x2
 2   3   4      4    3     5
rectangular triangle 4-3-5
periphery 3x4 = 12
area 1x1x2x3 = 6
```

radius of the inscribed circle 1x1 = 1
diameter 1x2 = 2
tangents of the half angles 1, 1/2, 1/3

```
2   3   4       4x5  3x7  2x2+5x5
 5   7   10    20   21     29
```

rectangular triangle 20-21-29
periphery 7x10 = 70
area 2x3x5x7 = 210
radius of the inscribed circle 2x3 = 6
diameter 3x4 = 12
tangents of the half angles 1, 2/5, 3/7

```
5    7    10       10x12  7x17  5x5+12x12
 12   17    24      120   119     169
```

rectangular triangle 120-119-169
periphery 17x24 = 408
area 5x7x12x17 = 7140
radius of the inscribed circle 5x7 = 35
diameter 7x10 = 70
tangents of the half angles 1, 5/12, 7/17
and so on

```
2   7    4       4x9  7x11  2x2+9x9
 9   11   18    36   77      85
```

rectangular triangle 36-77-85
periphery 11x18 = 198
area 2x7x9x11 = 1386
radius of the inscribed circle 2x7 = 14
diameter 7x4 = 28
tangents of the half angles 1, 2/9, 7/11

```
9    11    18       18x20  11x29  9x9+20x20
 20    29    40     360    319     481
```

rectangular triangle 360-319-481
periphery 29x40 = 1160
area 9x11x20x29 = 57420
radius of the inscribed circle 9x11 = 99
diameter 11x18 = 198
tangents of the half angles 1, 9/20, 11/29
and so on

The same number patterns generate quadruples:

1 1 2 1x2=2 1x1=1 1x2=2 1+2=3 quadruple 2-1-2-3
2 3 4 3x4=12 3x3=9 2x4=8 9+8=17 quadruple 12-9-8-17
5 7 10 7x10=70 7x7=49 5x10=50 49+50 quadruple 70-49-50-99

```
2 7 4  7x4=28  7x7=49  2x4=8  49+8=57  quadruple 28-49-8-57
9 11 18  11x18 11x11 9x18 (11x11)+(9x18)  q 198-121-162-283
20 29 40  29x40 29x29 20x29 (29x29)+(20x29)  1160-841-800-1641
```

All is equal, all unequal … (new version)

a paper for the Brno conference 2003, Chairman Alan Rogerson; *The Decidable and Undecidable in Mathematics Education, a tribute to Kurt Goedel*

Mathematical logic

As a teenager I read the first sixty pages of a book on quantum mechanics without understanding anything, but I was fascinated by a footnote which said that the basic equation of mathematics *a* = *a* has not yet been thoroughly studied. Finally something I could understand! A couple of years later I found the following explanation. Mathematics is based on the formulas

a = a   a unequal to b

which represent the logic of building and constructing. Consider these examples:

b = b = b = b = b = ...

A wall can easily be built and will stand firmly if the bricks (b) are of the same material, have the same size, and consistency, and have in common all other properties; in short: if they are equal.

b = b

The bricks should neither soak up moisture nor crumble in the rain, nor crack in the summer heat; their properties must remain stable, they must stay as they are.

2 + 1 + 3 = 6 = 2 + 1 + 3

A closed door (1) should become a part of the wall (6); one might then wish to open the door (2 + 1 + 3).

0.999... = 1

A door (0.999...) must fit into its frame (1), otherwise it will be too tight, or there will be a draft.

9 = 2 + 3 + 4 = 9

In order to clean or repair a machine (9) one dismantles it into single components (2, 3, 4); then one reassembles the parts, in order to return the machine to its former functioning state (9).

Sooner or later, a mathematical discovery finds its way into technology. The imaginary number *i* (square root of minus one) was first regarded as a strange number, yet without that funny little number no radio, television set or computer would work today.

The logic of nature, life and art

The logic of nature, life and art is based on another formula:

All is equal, all unequal ...

(Johann Wolfgang von Goethe, *Wilhelm Meisters Wanderjahre, Aus Makariens Archiv*)

a = a

An apple is an apple; yet one fruit may be red and sweet, another green and sour, and another yellow and juicy ...

A rose is a rose is a rose (Gertrude Stein)

You may imagine a red rose, a white rose, a sweet smelling yellow tea rose, a budding rose, a flowering rose, or you may think of a girl named Rose, Rosy, Rosemary.

A snowflake is a snowflake

Each snowflake forms a hexagon, yet seen under a microscope every flake has its own unique pattern.

A mouse is not an elephant

Mice and elephants belong to the animal kingdom, and are mammals; they have a common mouse-like ancestor with a kind of proboscis, while the hyrax, the elephant's nearest living relative, resembles a large mouse.

We are all equal and all different

A fair and reasonable human society is based on the fundamental equality of all humans, while leaving room for our individualities

p = p = p = p = p = p = ...

Physicists search for elementary particles that fulfill the above equation. They explore and expand the realm of technology, without end.

In his *Diary of the Italian Journey*, Goethe speaks of an *ever turning key*. This key might well have been the formula *All is equal, all unequal,* which Goethe successfully applied to the morphology of plants and animals, and also to works of art. Here is a wonderful quote from a later essay on the fine arts, given in the original German: *Alles, was uns daher als Zierde ansprechen soll, muss gegliedert sein, und zwar im höheren Sinne, dass es aus Teilen bestehe, die sich wechselsweise aufeinander beziehen. Hiezu wird erfordert, dass es eine Mitte habe, ein Oben und Unten, ein Hüben und Drüben, woraus zuerst Symmetrie entsteht, welche, wenn sie dem Verstande völlig fasslich bleibt, die Zierde auf der geringsten Stufe genannt werden kann. Je mannigfaltiger dann aber die Glieder werden, und je mehr jene anfängliche Symmetrie, verflochten, versteckt, in Gegensätzen abgewechselt, als ein offenbares Geheimnis vor unsern Augen steht, desto angenehmer wird die Zierde sein, und ganz vollkommen, wenn wir an jene ersten Grundlagen dabei nicht mehr denken, sondern als von einem Willkürlichen und Zufälligen überrascht werden.*

A key episode from my school days

Let me recall a key episode from my school days. A teacher told us that we are forbidden to divide a number by zero. I replied that I could carry out this calculation by choosing ever smaller divisors, until, finally, I obtained an infinitely large number, and that when I would then multiply this number with zero I would obtain 1 and any other number. My teacher took hold of his iron key and knocked me on the head. When I repeated my arguments in the next lesson he knocked me on the head again. Thus I came to learn that even mathematics, the kingdom of pure logic, has its forbidden zones, where logic fails and iron keys are required.

In the light of the above insights, I shall put it like this: the division of a number by zero is a case where we leave the realm of mathematical logic in favor of the realm of art, life and nature, where all is equal and all unequal. Mathematical logic is a mental tool of building and constructing. If we wish to have this tool ready to hand we are not allowed to divide a number by zero, or to carry out similar operations, but we are always allowed to investigate the reverse side of the mirror and to explore the other realm of logic.

Language

Once I was invited to give private lessons to a boy who had difficulties understanding the concept of equations. I told him: let us put away the schoolbooks and instead have a look at language. I asked him to give me an example of a sentence, and he did so. I then showed him that he just formulated a verbal equation. As I don't recall his example, I shall invent a new one:

The football game will take place tomorrow.

This sentence contains an equation:

the football game - is - a tomorrow's event

We considered other sentences, and they always contained one or several equations.

A car passes by / a car - is - something that passes by

We see a movement, we recognize a car, and the car and the act of moving belong together, allowing us to formulate an equation.

The flowers are blooming.

We see a happy sway of colors, we identify it with plants, and we formulate an equation.

Language is the means of winning the help and care of others on whom we depend in one way or another. By speaking we are building a world into which we invite our listener, and we shape and color it in such a way as to please him and to win his help and support. In order to do so we require a mathematical logic that is present in the basic structure of a sentence. All children, even those not good at mathematics, can establish this ‘verbal equation’ quite easily and naturally. It may well be that many children who fear mathematics are capable of building the best formulated, most elegant and colorful sentences.

A further solace

A further solace for those who are less proficient at mathematics: there are two realms of logic: a mathematical one; and another - a treasury of mathematical and scientific laws which have not yet been discovered. Those good at mathematics may be clever in a limited way, in the realm of known laws, while others may be clever in handling the many still unknown laws that nevertheless rule our lives. Dear teachers, you know well that half of your work involves motivating your pupils, and that the best way to do this is to recognize and name the various abilities and talents of each individual child.

All is equal, all unequal … (old version)

Mathematical logic is based on the well-known formulas

a = a

and

a unequal b

Equations can be regarded as technical situations:

b = b = b = b = b = ...

A wall can easily be built and will stand firmly if the bricks (b) are of the same material, have the same size, consistency, and have in common all other properties; in short: if they are equal.

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The bricks should neither soak nor crumble in the rain, nor crackle in the summer heat; their properties must be stable, they must remain as they are.

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In order to clean or repair a machine (9) one wishes to dismantle it into single components (2, 3, 4); then one will want to reassemble the parts, in order to return it to its former well-functioning state (9).

Sooner or later, a mathematical discovery finds its way into technology. The imaginary number i (square root of minus one) was first regarded as a strange number, yet without that funny little number no radio, television set or computer would run today.

Looked at the other way around, an achievement such as building a pyramid would not have been possible without mathematical knowledge.

The logic of nature, life and art is based on another formula:

All is equal, all unequal ...

(Johann Wolfgang von Goethe, Wilhelm Meisters Wanderjahre, Aus Makariens Archiv)

a = a

An apple is an apple; yet while one fruit may be red and sweet, another may be green and sour, and another yellow and juicy ...

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Culture as a task: integrating man-made things into our lives and natural surroundings, utilizing them in the best possible way, minimizing their risks, preventing their abuse, developing suitable forms of life.

Art: human measure in a technical world.

May I add my definition of language? We depend on others and their manifold abilities, and we convey our needs and wishes we are unable to satisfy alone by means of language: facial expressions, signs, body language, sounds and words. Using words we describe and explain our situation in such a way as to gain the understanding, esteem, sympathy and help of our listeners. The more manmade things we use the more complex and specialized our lives become, and the more words we require. Yet every means of conveying needs and wishes in order to obtain help from others may be regarded as language.

While the mathematical formulas are the core of technology, a work of art conveys a more human measure.

While the natural sciences rely on mathematics and often generalize a successful insight, the humanities rely on art and keep alive our sense of the manifold aspects of life and nature.

Mechanical devices and machines have been very successful, and the mechanical paradigm has been generalized: the cosmos regarded as a clockwork mechanism, animals seen as mere automata, incapable of feeling.

While the natural sciences are prone to generalize a successful insight and ignore other, no less important aspects of life and nature, the humanities keep alive a sense of complexity.

Some female figurines from predynastic Egypt anticipate the Greek goddess Gaia. The idea of a biosphere, a kind of living earth, came to the fore again in the work of James Lovelock and Lynn Margulis.

The ancients regarded the human being as a tiny cosmos in a living universe represented by various deities. Are modern views so very different from ancient? Fractal geometry allows the same form to appear on every scale, while we consider the principle of life to be present in the universe or in matter (Carl Sagan).

Matter and energy are equivalents (Albert Einstein). May the same hold for energy and information? When we carry out a task in a clever way we require less energy. Might 'intelligent' matter from a new physics eventually circumvent the second law of thermo- dynamics? (an old fantasy of mine)

Silly questions of mine, formulated in my own personal freestyle English (on which I hold a copyright, so please no one dare copy my mistakes ;-) On which level does life emerge? on the level of molecules? or may there be a deeper organization of living matter? Is an electron in a stone really the same as an electron in a living cell? If there should be a subatomic organization of life, how deep would it reach? where does life begin? A philosophical principle of mine: no statement is absolutely true, and none absolutely wrong. Spirit: innermost order of matter?